Chaos, Solitons and Fractals 112 (2018) 103–115

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Pricing and hedging vulnerable option with funding costs and collateral

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ARTICLE INFO

Article history:
Received 8 January 2018
Revised 4 April 2018
Accepted 30 April 2018

JEL classification:
G13
D81

MSC:
91G20
91G40
91G60

Keywords:
European vulnerable option
Funding spreads
Collateral
Local volatility
Backward stochastic differential equations

ABSTRACT

We explore the valuation and hedging strategies of a European vulnerable option with funding costs and collateralization for local volatility models. It is found that, in the absence of arbitrage opportunities, the option price must lie within a no-arbitrage band. The boundaries of no-arbitrage band are computed as solutions to backward stochastic differential equations (BSDEs in short) of replicating strategy and offsetting strategy. Under some conditions, we obtain the closed-form representations of the no-arbitrage band for local volatility models. In particular, the fully explicit expressions of the no-arbitrage band for Black–Scholes model and the constant elasticity of variance (CEV) model with time-dependent parameters are derived. Furthermore, we provide a strategy for the option holder by using the risky bond issued by the option writer to hedge the remaining potential losses. By virtue of numerical simulation, the impact of the default risk, funding costs and collateral can be observed visually.

1. Introduction

Over-the-counter (OTC in short) markets have grown rapidly in recent years. Default risk in OTC transactions has attracted special attentions since the global financial crisis in 2008, as evidenced by the collapse of Lehman Brothers. Unlike the options traded on regulated exchanges, holders of OTC options are exposed to potential credit risks due to the possibility of their counterparties being unable to fulfill the necessary payments on the exercise dates. As a result, option holders require their counterparties to post collateral for reducing the default risk exposure.

Taken from Bielecki and Rutkowski [5] Section 1.2, a European vulnerable option is an option contract in which the option writer may default on his obligations. In other word, this is an option whose payoff at maturity depends on whether a default event, associated with the options writer, has occurred before or on the maturity, or not. The default risk of the holder of the option is manifestly not relevant.

One method to price European vulnerable options introduced by Klein [23] is similar to the firm value approach. The structural approach was first introduced in Merton [29], where a single-period model was utilized to derive the default probability from the random variation in the unobservable value of the firm's assets. Following the framework of Klein [23], there are a lot of literatures to study the vulnerable option pricing in different environments, such as Hung and Liu [22], Tian et al. [37], Lee et al. [26], Wang [38], Han [21] and so on. In this method, default event is specified in terms of the evolution of the total value of the counterparty firm's assets as well as in some terms of some default triggering barrier. One of the major shortcomings of the firm value approach lies in the presumption that the firm value cannot be directly observed. The other major class of credit risk modeling research focuses on reduced-form models of default, which assumes a firm's default time is inaccessible or unpredictable and driven by a default intensity that is a function of latent state variables (cf., e.g., Litterman and Iben [27] and so on). Due to their mathematical tractability, these models have become very popular amongst practitioners. Lando [25] presented a framework for modeling defaultable securities and credit derivatives which allows for dependence between market risk factors and credit risk. For European vulnera-

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https://doi.org/10.1016/j.chaos.2018.04.042
0960-0779/

ble options, Fard [19] formulated the credit risk in a reduced form model and the dynamics of the spot price in a completely random generalized jump-diffusion model. However, they do not consider the impact of collateral in their models. In this work, we will consider the reduced form model with collateral, furthermore, we consider the impact of funding rates spreads for pricing and hedging OTC options.

The classical pricing formula of European options is based on the no-arbitrage argument and replicating strategy of the option. However, when implementing the strategy in actual market, the trader is required to raise cash in order to finance a number of operations. Those include maintaining the position of the hedge, posting collateral resources, and paying interest on collateral received. If the trader is borrowing, he will be charged an interest rate depending on current market conditions as well as on his own credit quality. Such a rate is typically higher than the rate at which the trader lends excess cash. The difference between borrowing and lending rate is also referred to as funding spread.

Without default risk, there are a lot of literatures that studied the claims pricing in markets with differential rates. Bergman [3] considered option pricing in a market with higher borrowing than lending rate and derived an arbitrage-band where the option price must lie within. Korn [24] also provided a similar result. Piterbarg [34] developed Black–Scholes pricing formula with credit support annex (CSA in short) of the International Swaps and Derivatives Association master agreement. Backward stochastic differential equations (BSDEs in short) theory plays an important role in studies of self-financing trading strategy with funding costs. El Karoui et al. [18] studied the superhedging price of a contingent claim under rate asymmetry via nonlinear BSDEs. In a general semimartingale market framework, Bielecki and Rutkowski [6] derived the BSDE representation of the wealth process associated with a self-financing trading strategy with funding costs and collateralization. For default risk, there are a lot of literatures to study the valuation of claims with bilateral counterparty risk, such as Brigo and Pallavicini [7], Burgard and Kjaer [12], Burgard and Kjaer [13], Nie and Rutkowski [30], Pallavicini et al. [31], Pallavicini et al. [32]. Crépey [14,15] introduced a BSDE approach for the valuation of counterparty credit risk by taking funding constraints into account. Bichuch et al. [4] developed a framework for computing the total valuation adjustment of a European claim accounting for funding costs, counterparty credit risk, and collateralization. Brigo et al. [8–11] made a lot of contributions for funding valuation adjustment. Brigo et al. [10] studied the nonlinear valuation equations for a consistent framework including CVA, DVA, collateral, netting rules and re-hypothecation. Brigo et al. [11] we develop a risk-neutral pricing formula for consistent valuation of collateralized as well as uncollateralized trades under counterparty credit risk, collateral margining, and funding costs.

In this paper, we consider the valuation and hedging of European vulnerable options including the impact of funding spreads and collateral. The underlying of an OTC option is a default-free stock and the risky bond issued by the counterparty is used to hedge the default risk. At first, we construct two simulation strategies, the replicating strategy and offsetting strategy. The wealth processes of the replicating strategy and offsetting strategy are following as some nonlinear BSDEs. The replicating strategy is used to determine the upper bound of the option price with the no-arbitrage principle. The lower bound of the option price is determined by the offsetting strategy with the no-arbitrage principle. Under some conditions, we can prove that the trader only need borrowing rate for constructing replicating strategy and lending rate for constructing offsetting strategy. In these cases, the nonlinear BSDEs of the replicating strategy and offsetting strategy reduce to linear BSDEs, such that the solutions of these BSDEs have explicit expressions. It is worth noting that the simulation strategies are not suitable for the actual hedging demands of the option writer and option holder. Therefore, we provide a strategy for the option holder to hedge counterparty risk and a strategy for the option writer to hedge the option payoff and post collateral. We employ numerical simulations of Black–Scholes model and CEV model to show the impact of the funding costs.

The rest of this paper is organized as follows. In Section 2, we develop the notations for market model considered in this paper. In Section 3, we introduce the replicating strategy and offsetting strategy of vulnerable European options. Furthermore, we supply the explicit expressions for no-arbitrage interval under some conditions. Hedging strategies for option writer and option holder are studied in Section 4. We present numerical simulations for Black–Scholes model and CEV model in Section 5. Section 6 provides a conclusion.

2. Model

We consider a probability space $\{\Omega, \mathcal{F}, \mathbb{P}\}$ to model the financial market. Here, $\mathbb{P}$ is the physical probability measure. In this paper, we consider a European vulnerable option, this means that the option holder payed the option premium to the option writer at the beginning of the option contract. And the option writer posted collateral to the option holder in the period of the option contract. The option holder only has rights and no obligations. The option writer only has obligations and no rights. Hence, we only consider the default risk of the option writer.

We let that $\mathfrak{F} = (\mathcal{F}_t)_{t\geq 0}$ is completion and argumentation of the filtration generated by the Brownian motion $W^{\mathbb{P}}$ under the measure $\mathbb{P}$. We assume that $\mathfrak{F}$ contains all market information except for default events. The filtration containing default event information is denoted by $\mathfrak{H} = (\mathcal{H}_t)_{t\geq 0}$. Here, $\mathfrak{H}$ is generated by the default time τ_C of the option writer. The filtration $\mathfrak{G} = (\mathcal{G}_t)_{t\geq 0}$ is the filtration $\mathfrak{F}$ progressively enlarged by τ, i.e., $\mathfrak{G} = \mathfrak{F} \vee \mathfrak{H}$.

Let us introduce the notation for market models considered in this paper.

- Funding accounts

The cash account B^f is used for unsecured lending or borrowing of cash. In the case when the borrowing and lending cash rates are different, we use symbols r_f^l (resp., r_f^b) to denote the lending (resp., borrowing) rate. Hence the superscripts l (resp., b) will refer to rates applied to deposits (resp., loans) from the viewpoint of the funding account of the trader. Let ψ_t^f be the number of shares of the funding account at time t. Define

$$B_t^f \triangleq \exp\left\{\int_0^t r_f(\psi_s^f)ds\right\}, \tag{2.1}$$

where $r_f(x) = r_f^l I_{\{x>0\}} + r_f^b I_{\{x<0\}}$. If the trader's position ψ_t^f is negative, then he needs to finance for maintaining his position. He will do so by borrowing cash amount at the rate r_f^b. Similarly, if the ψ_t^f is positive, he will lend the cash amount at the rate r_f^l.

Under normal circumstances, the borrowing rate r_f^b is larger than the lending rate r_f^l, i.e., $r_f^b \geq r_f^l$. This will lead to funding costs of hedging strategies and trading strategies.

- The underlying stock security

We denote by S the price of the underlying stock. Under the physical measure, the dynamic of the stock price is given by local volatility model as follows,

$$dS_t = \mu(S_t, t)S_t dt + \sigma(S_t, t)S_t dW_t^{\mathbb{P}}, \tag{2.2}$$

where μ and σ are determinate functions denoting, respectively, the appreciation rate and the volatility of the stock.