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Analysis of limit cycles and stochastic responses of a real-power vibration isolation system under delayed feedback control

Dongmei Huang*, Wei Li, Guidong Yang, Meijuan He

School of Mathematics and Statistics, Xidian University, Xi'an, Shaanxi 710071, China

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ABSTRACT

In this paper, the dynamical properties of a real-power vibration isolation system with delayed feedback control subjected to deterministic and stochastic excitations are considered. According to the free vibration analysis, it is found that a large number of limit cycles may be existed for certain time delay and feedback gain. Then, the relationship of amplitude and frequency is derived for the undamped system. For the system with harmonic excitation, multi-valued phenomena are observed due to the existence of the limit cycles. In this respect, with the change of time delay, in every period the response is similar to time delay island, and the number of islands is different under different excitation frequency. Additionally, for analyzing the complex dynamic properties, the vibration isolation system with Gauss white noise excitation is explored by the largest Lyapunov exponent and the stationary probability density. The symmetrical period-doubling bifurcation phenomenon is found and verified. Finally, by using Monte Carlo simulation, the stationary probability density is explored from original system. The change of time delays can induce the occurrence of stochastic bifurcation and the response from two peaks becomes triple peaks.

1. Introduction

The force-deflection relationship of the elastic elements is usually modelled as polynomials which are appropriate in a limited range of the deflection or just valid in some intervals of the parameter values [1,2]. However, the deflection properties with the positive integer-order powers could not exactly describe the force-deflection relationship of some materials, such as the compressed coils, the flexible elements, and the open-celled foam. Recently, the non-polynomial deflection characteristic is developed to describe the nonlinearity of these specialized material [3–5], which provides a comprehensive and thorough understanding of the material nonlinearity. Actually, the vibration isolation system with non-polynomial nonlinearity has received increasing interest. For examples, Kovacic with her co-authors has done considerable work about this kind of system [3–6]. The dynamical properties of a QZS vibration isolator were analyzed in Ref. [4]. And by applying an elliptic averaging method, the performance characteristics of the relative and absolute displacement transmissibility were discussed in Ref. [5]. Then, the time responses of a chain of two-mass oscillators, whose power of nonlinearity can be any real number higher than unity, were determined in exact analytical forms in Ref. [6]. Subsequently, the steady state response and transmissibility of vibration isolators with symmetric as well as asymmetric restoring forces were considered by Ravindra and Mallik [7]. Additionally, the Melnikov procedure in the system with fractional order deflection function, which can define the criteria for transversal intersection of the stable and unstable manifolds, was discussed by Cveticanin and Zukovic [8].

Obviously, the previous study about the vibration isolation system mainly focused on the dynamical analysis. However, the instability of the systems might imply excessive vibration and even damages to the system, thus, the effective vibration control strategy is indispensable and practicable for improving the system stability and controlling the corresponding performance. In past several decades, numerous control strategies have been used in the dynamic systems, such as adaptive sliding control [9], fuzzy control [10], proportional-integral-derivative (PID) control [11], and NMPPF control [12]. Particularly, in the study of dynamical system with active control, due to the limitation of research methods, most of them ignored the effect of time delay on the control signal. But as a special kind of physical phenomenon, time delay, which plays an important role in analyzing and controlling the dynamical response of the system, is usually unavoidable in feedback control systems due to time spent in measuring and estimating the system state, calculating and executing the control forces, etc.

* Corresponding author.
E-mail address: dmhuang@xidian.edu.cn (D. Huang).

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Reasonably introducing and processing time delay usually can improve the performance of the system effectively [13–20]. On one hand, the existence of time delay can result in rich dynamical behaviors such as frequency island, multi-valued response [15], Hopf bifurcation [16,17], quasi-periodic motion [18], jump and hysteresis phenomena [19]. For example, the multi-response characteristics in the vibration isolation system with real-power nonlinearity under time delay feedback control were discussed by Huang et al. [15]. The existence of Hopf bifurcation of a food-limited population model when delay passes through a sequence of critical values was found by Wei et al. [16,17]. On the other hand, the time delay feedback control is to be proposed for suppressing the vibration and controlling the stability of the primary system. Based on this idea, delayed feedback control was used to suppress the vibration in a torsional vibrating system by Zhao et al. [18]. The state feedback control with time delay was applied to suppress the principal resonance of a parametrically excited Liénard system by Maccari [19]. Moreover, in order to control the dynamical behavior of a nonlinear electromechanical coupling system, time delay feedback was introduced in this system and the static bifurcation characteristic and stability were studied by Liu et al. [20].

It can be seen that a lot of endeavors have been devoted to investigate the stability and performance of the deterministic dynamic system with time delay feedback control, however, because of the uncertainty inherent not only in system parameters, but also in external excitations [21], considering the dynamical property with respect to dynamical stability of stochastic controlled systems is of paramount importance. In fact, in the past decades, extensive developments have been studied [22–25]. The stability of the stationary solution by the method of Lyapunov exponent was investigated by Grigoriu [22]. Recursive formulas in terms of statistical response of linear systems with time delay under normal white noise input were developed by Di Paola [23]. By using the method of multiple scales, the principal resonance of a Duffing oscillator with delayed state feedback under narrow-band random parametric excitation was studied by Jin et al. [24]. By means of the Melnikov technique, the chaotic behavior of a double-well Duffing oscillator with both delayed displacement and velocity feedbacks under harmonic excitation was investigated by Sun et al. [25]. Nevertheless, the stochastic dynamical response of the vibration isolation system with real-power restoring force under delayed feedback control, to authors' knowledge, has not been reported in literatures.

Dynamical responses of the vibration isolation systems with nonlinear restoring force described by the force-deflection relationship with the non-integer order, where the randomness in external excitations or parametric excitations is involved, can give us a comprehensive and desired understanding of the controlled system performance. Motivated by the above findings, this study aims at gaining an insight into the nonlinear dynamic behavior from deterministic and stochastic cases, respectively.

The paper is organized as follows: firstly, based on free vibration analysis, the number of limit cycles is presented. Then, the responses of the system with damping and external excitation are discussed. Furthermore, by means of the maximum Lyapunov exponent, the dynamical behavior is analyzed. The stationary probability density is discussed by means of the Monte Carlo simulation method. Effects on the stationary probability density for different damping coefficients, time delays and noise intensities are discussed. Concluding remarks are included in the final section.

2. Free vibration analysis with damping

In the first step of analyzing the dynamical behavior of the vibration isolation system without external excitation under time delayed feedback control, the corresponding differential equation of

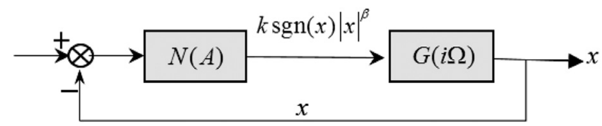

Fig. 1. The block diagram of the system.

the system is given by Eq. (1),

$$\ddot{x} + d\dot{x} + x + k\text{sgn}(x)|x|^{\beta} = g_1\dot{x}(t-\tau) + g_2 x(t-\tau), \tag{1}$$

where the fourth term on the left side stems from the restoring force and has a single-term power form, k is positive, β is the power which could be any non-negative real number. Dots denote the derivatives with respect to the non-dimensional time t, x is the non-dimensional displacement, d is linear damping coefficient. On the right side, τ is the inherent system delay, g_1 and g_2 are the corresponding feedback gains.

Then, the equation can be rewritten as

$$\ddot{x} + d\dot{x} + x - g_1\dot{x}(t-\tau) - g_2 x(t-\tau) = -k\text{sgn}(x)|x|^{\beta}, \quad \beta \neq 1 \tag{2}$$

which is represented as a block diagram as shown in Fig. 1 following the form of the describing function analysis. Denoting the function on the right side of Eq. (2) as the forcing function $f(t)$, then, the transfer function of the linear system should be

$$G(z) = \frac{1}{z^2 + dz + 1 - g_1 s e^{-z\tau} - g_2 e^{-z\tau}}, \tag{3}$$

where $z = i\Omega$ is the complex frequency variable ($i = \sqrt{-1}$). Herein, the approximation solution of Eq. (3) is assumed as $x = A\sin(\Omega t)$.

The corresponding characteristic function of the nonlinear system should be [26]

$$1 + N(A)G(i\Omega) = 0, \tag{4}$$

where $N(A) = \frac{2^{\beta+2}kA^{\beta-1}}{\pi}B(\frac{\beta+2}{2}, \frac{\beta+2}{2})$.

For separating the imagery and real part, denoting

$$1 + N(A)G(i\Omega) = P + iQ. \tag{5}$$

Then, according to Eqs. (3)–(5), it is easy to obtain

$$P = \frac{2^{\beta+2}kA^{\beta-1}}{\pi}B\left(\frac{\beta+2}{2}, \frac{\beta+2}{2}\right) - \Omega^2 + 1 \\ - g_1\Omega\sin(\Omega\tau) - g_2\cos(\Omega\tau), \tag{6}$$

and

$$Q = d\Omega - g_1\Omega\cos(\Omega\tau) + g_2 sin(\Omega\tau). \tag{7}$$

Further, based on Eqs. (4) and (5), it should have

$$P = 0 \text{ and } Q = 0. \tag{8}$$

Assuming that A_0 and Ω_0 are the solutions of Eq. (4), then, define a function Φ and if the following equality

$$\Phi = \frac{\partial P}{\partial A}\frac{\partial Q}{\partial \Omega} - \frac{\partial P}{\partial \Omega}\frac{\partial Q}{\partial A} > 0, \tag{9}$$

is satisfied, the solutions are stable, otherwise, they are unstable. That is

$$\frac{2^{\beta+2}k(\beta-1)A^{\beta-2}}{\pi}\mathrm{B}\left(\frac{\beta+2}{2}, \frac{\beta+2}{2}\right)[d - g_1\cos(\Omega_0\tau) \\ + g_1\Omega\tau\sin(\Omega_0\tau) + g_2\tau\cos(\Omega_0\tau)] > 0. \tag{10}$$

For the case of $\beta > 1$, it has

$$d - g_1\cos(\Omega_0\tau) + g_1\Omega\tau\sin(\Omega_0\tau) + g_2\tau\cos(\Omega_0\tau) > 0, \tag{11}$$