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State estimation of chaotic Lurie system with logarithmic quantization

Juanhui Zheng[a,b,*], Baotong Cui[a,b]

[a] Key Laboratory of Advanced Process Control for Light Industry (Ministry of Education), Jiangnan University, Wuxi 214122, China
[b] School of IoT Engineering, Jiangnan University, Wuxi 214122, China

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ABSTRACT

In this paper, we address the problem of state estimation of the Lurie system via the communication channel in the case of only this system outputs available. A coder-decoder scheme combines with a logarithmic quantization to form a novel and reliable communication channel. The errors between Lurie system outputs and observer outputs are regarded as the feedback signals, which are transmitted into the observer though the communication channel. A sufficient condition for input-to-state stability is given for the boundedness of the error of state estimation. The results of two examples show the effectiveness and superiority of the proposed communication channel of the logarithmic quantization.

1. Introduction

Lurie system is a classical uncertain nonlinear system which is a constant focus of research, such as absolute stability [1,2], chaotic synchronization [3,4] and secure communication [5–7]. Recently, the robust performance analysis for chaotic Lurie nonlinear control systems have been investigated [8,9]. At the same time, a variety of the synchronization problem of chaotic Lurie complex dynamical networks and passifiable Lurie systems have attracted general attention [10–12], such as sampled-data synchronization problems [7,13–15] , impulsive synchronization problems [16,17] and master-slave synchronization problems [18,19]. Through the description above, as is known to us, most researches of chaotic Lurie system are based on the assumption that states of Lurie system are known. However, in practice, states of Lurie system are not measurable so that the problem of state estimation is important for the Lurie system.

Since state estimation is an important problem in the internal dynamic law of systems, what is more, it is also significant prerequisite for the control of systems. More recently, the problems of state estimation have attracted increasing attention. The majority of state estimation problems are studied for linear systems [20–22] and neural networks [23,24]. The primarily approaches of state estimation are the least square estimate and the Kalman filtering for linear systems. However, methods of state estimation for nonlinear systems are not enough. Then, State estimation of nonlinear Lurie system is firstly put forward by constructing a state observer with the limited communication channel in [26]. Motivated by this research [26], we can establish the novel and available communication channel to estimate accurately.

As we all know, the combination of communication and control theory is a hot research area. The state observer is controlled by the feedback signals which are transmitted though the code-decoded quantization scheme so that the feedback signals can be transmitted remotely at the limited communication channel. In order to simplify the feedback process, we usually assume there is not any channel distortions and disturbances in communication channel [12,25–28].

With respect to quantization method, it has attracted a growing interest for many researchers. The quantized feedback stabilization for the continuous- and discrete-time linear systems and nonlinear systems have been analyzed with saturating quantized measurements [29]. Regarding to quantization feedback control and quantization state estimation about communication networks, Fu [30] has explained the advantages and disadvantages about the linear quantization, logarithmic quantization, nonlinear quantization and dynamic quantization in the area of quantization feedback control and estimation. The logarithmic quantization which can be divided into the infinite logarithmic quantization and truncated logarithmic quantization has attracted more attention on networked control system [31–35]. In recent years, the uniform static quantizer, time-varying coder with a memory and adaptive coder have been involved in the limited-band communication channel for synchronization and state estimation of nonlinear systems [12,26–28,36,37].

* Corresponding author at: Key Laboratory of Advanced Process Control for Light Industry (Ministry of Education), Jiangnan University, Wuxi 214122, China.
E-mail address: 6161918007@vip.jiangnan.edu.cn (J. Zheng).

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This paper is inspired by the researches of [26] and [30]. The principal difference comparing with [26] is that the logarithmic quantizer is applied to quantize feedback signals during the processing of coding and decoding and firstly form a feasible novel communication channel with the logarithmic quantizer. The rest of the paper is organised as follows. In Section 2, the Lurie system and the state estimation strategy are described via limited communication channels. In Section 3, a logarithmic quantization method is employed to deal with the transmission of the feedback output error signal. In Section 4, the state estimation errors dynamic function is studied based on the input-to-state stability method for the proposed systems. In Section 5, the availability and superiority of the communication channel with proposed quantization code-decode procedure are substantiated in the Chua!'®s circuit and chaotic Chua system. In Section 6, summaries of the full paper and future research are proposed.

2. State estimation scheme

Consider the Luire system model, which consists of the linear and nonlinear part, described as follows:

$$\dot{x}(t) = Ax(t) + B\varphi(y(t)), \quad t \geq 0, \tag{1}$$

$$y(t) = Cx(t), \tag{2}$$

where $x(t) = (x_1, x_2, \dots, x_n)^T \in \mathbb{R}^n$ is the indirect measurable state vector of system; $y(t) \in \mathbb{R}$ is the measurable output scalar of system; $\varphi(y)$ is the continuous nonlinear scalar function; $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times 1}$, and $C \in \mathbb{R}^{1\times n}$ are constant matrices.

In order to estimate the unmeasurable states $x(t)$ of the Lurie system, a state observer of full order has been constructed as follows:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B\varphi(\hat{y}(t)) + L\bar{\varepsilon}(t), \quad t \geq 0, \tag{3}$$

$$\hat{y}(t) = C\hat{x}(t), \tag{4}$$

where $\hat{x}(t) = (\hat{x}_1, \hat{x}_2, \dots, \hat{x}_n)^T \in \mathbb{R}^n$ is the observer state vector of estimation; $\hat{y}(t) \in \mathbb{R}$ is the observer output scalar; $L \in \mathbb{R}^{n\times 1}$ with a scalar gain κ is designed by:

$$L = \kappa B, \tag{5}$$

where $B \in \mathbb{R}^{n\times 1}$ is a matrice.

Then $\varepsilon(t)$ denotes the output error of the system and observer:

$$\varepsilon(t) = y(t) - \hat{y}(t) = Ce(t), \quad t \geq 0, \tag{6}$$

where $e(t) = x(t) - \hat{x}(t)$ is the state estimation error.

The output error signals $\varepsilon(t)$ of the above-mentioned plant and observer are transmitted directly into the observer. However, we propose the state observer whose input feedback signals have been transmitted via the encoder and decoder, which make decisions based on the same information under a communication channel, such as [12,26]. We suppose that the communication channel is ideal without the transmission delay and the channel interference.

The transmission process of output error signals $\varepsilon(t)$ is plotted in Fig. 1 and depicted specifically as follows.

Firstly, $\varepsilon(t)$ should become discrete values at the consistent sampling time instant for coding.

$$\varepsilon[k] = \varepsilon(t_k), \quad t_k = kT, \quad k = 0, 1, 2, \dots, \tag{7}$$

where T is the sampling period.

After sampling, the discrete feedback signal are coded by employing a coding quantizer $Q(\cdot)$. In this work, different from [26], we take the classical logarithmic quantizer which has been rarely considered in the communication channel into account and the novel communication channel with the logarithmic quantizer is proposed. The method of logarithmic quantization has been given in details in the next section,

$$\bar{\varepsilon}[k] = \bar{\varepsilon}(t_k) = Q(\varepsilon(t_k)), \quad t_k = kT, k = 0, 1, 2, \dots. \tag{8}$$

For the decoding, the discrete quantization signal should be reverted to the continuous feedback signal, which serves as the new input of the state observer. $Zero-order\,extrapolation$ is utilized to make the discrete signal become the continuous signal [26],

$$\bar{\varepsilon}(t) = \bar{\varepsilon}[k], \quad t \in [t_k, t_{k+1}), \tag{9}$$

where the $\bar{\varepsilon}(t)$ is the ultimately feedback input signal.

The state observer (3) can be written as the form:

$$\dot{\hat{x}}(t) = A\hat{x}(t) + B\varphi(\hat{y}(t)) + L\bar{\varepsilon}(t), \quad t \geq 0. \tag{10}$$

The total transmission error $\delta_\varepsilon(t)$ of communication channel is defined:

$$\delta_\varepsilon(t) = \varepsilon(t) - \bar{\varepsilon}(t), \quad t \geq 0. \tag{11}$$

3. Logarithmic quantization

We consider the logarithmic quantizer as a discrete map $Q : \mathbb{R} \to \mathbb{R}$. A logarithmic quantizer is described as follows:

$$\nu = \{\mu_i = \rho^i \mu_0 : i = 0, \pm 1, \pm 2, \dots\} \cup \{0\}, \ \mu_0 > 0, \tag{12}$$

where $\rho \in (0, 1)$.

$$Q(y) = \begin{cases} \rho^i \mu_0, & if\ \frac{1}{1+\sigma}\rho^i\mu_0 < y \leq \dfrac{1}{1-\sigma}\rho^i\mu_0, \\ 0, & if\ y = 0, \\ -Q(-y), & if\ y < 0, \end{cases} \tag{13}$$

with the quantization density

$$\sigma = \frac{1-\rho}{1+\rho}. \tag{14}$$

A infinite-level logarithmic quantization process is depicted in Fig. 2 [30].

The proposed logarithmic quantizer has been compared with the time-based zooming memoryless binary quantizer [26] in the communication channel coder-decoder scheme. The binary quantizer is variable with the variation of time so that the quantization value is continuously decrease regardless of the magnitude of feedback signal, which leads to magnified quantization errors. The proposed logarithmic quantizer can overcome the disadvantage of the binary quantizer, since the quantization value increase or decrease with the variation of feedback signals so that quantization errors can be reduced. At the same time, in this paper, the feedback signal is not the output signal but the output error signal of system and observer which should be smaller than the output signal. Consequently, the small output error brings about the small quantization error and the relatively big output error generates the acceptable quantization error under the logarithmic quantizer.

According to the quantization (13), the coding quantized error signal (8) yields:

$$\bar{\varepsilon}[k] = \bar{\varepsilon}(t_k) = Q(\varepsilon(t_k)) = \begin{cases} \rho^i\mu_0, & if\ \dfrac{1}{1+\sigma}\rho^i\mu_0 < \varepsilon(t_k) \\ & \leq \dfrac{1}{1-\sigma}\rho^i\mu_0, \\ 0, & if\ \varepsilon(t_k) = 0, \\ -Q(-\varepsilon(t_k)), & if\ \varepsilon(t_k) < 0, \end{cases} \tag{15}$$

where $\rho \in (0, 1)$, $i = 0, \pm 1, \pm 2, \dots,$ $\mu_0 > 0$.

Remark 1. In this paper, the feedback signals $\varepsilon(t)$, which are the output error signals of system and observer, are transmitted