# Time-varying self-similarity in alternative investments

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## ARTICLE INFO

## ABSTRACT

A rolling-window based approach is adopted in the current work to gauge time evolution of long-range dependence in a large set of various alternative (nonconventional) markets including Islamic, sustainability, ecology, and ethical equity markets. The approach allows conveying important information about the dynamics of long-range dependence in such contemporary and trendy alternative investments. The study shows that all investigated "alternative" markets exhibit persistence/anti-persistence dynamics, except two which reveal pure random behaviour. Roughly half of the Islamic markets and all ecological markets demonstrate persistent dynamic behaviour. On the contrary, all sustainable and most of the ethical alternative markets show increased anti-persistence. The persistence/anti-persistence patterns observed are cyclical, namely they are clustered over time. Finally, the distributions of estimated Hurst exponents are statistically different within Islamic and non-Islamic alternative markets.

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## 1. Introduction

In econophysics, measuring long-range dependence is a common way to assess efficiency in international financial markets, including stock [1,2], bitcoin [3,4], commodity [5,6], and exchange markets [7], to name a few. Indeed, investigating market efficiency is crucial for portfolio and risk managers to achieve optimal allocation of financial resources and better returns. This work adds to studies examining market efficiency by looking at the time-evolution of long-range dependence in Islamic, sustainability, green, and ethical market indices. Undeniably, investing in such indices has become quite popular especially during the very recent years as they are considered alternative markets versus traditional/conventional ones. They incorporate a distinctive risk-return shape for investors and market makers.

A small number of works have examined statistically dynamical patterns for these alternative investments. Indicatively, a conditional heteroscedastic model was employed to assess long-memory in Malaysian Shariah Index [8], a fractionally integrated generalized autoregressive conditional heteroscedasticity was introduced to examine the long-memory features in Dow Jones Islamic Market World Index [9], whilst the efficiency of Islamic and conventional stock markets counterparts was cross-compared in [10] using MF-DFA. Moreover, the detrended fluctuation approach was applied to European Union Allowances (EUA) futures at the European Climate Exchange in [11], to generate long-range correlation patterns in carbon emission rights in futures markets of the Certified Emission Reduction. The EUA was also studied by means of MF-DFA and empirical mode decomposition as in [12], and multifractal detrended cross-correlation analysis based on maximum overlap wavelet transform was adopted in [13] to verify the presence of multifractality among carbon and energy markets.

Without a doubt, a number of questions remains unanswered vis-à-vis the existence of long-range dependence in alternative (nonconventional) investments, particularly in Islamic (Shariah), sustainability, ecology, and ethical markets. The first market category obeys to Islamic rules and guidelines, the second one follows long-term economic measurement, environmental and social criteria. The third category is committed to mitigating risks arising from climate change, and the fourth excludes companies that generate revenue from alcohol, tobacco, gambling, armaments and firearms, and/or adult entertainment. In this regard, we contribute to econophysics literature, by particularly aiming to investigate how long-range dependence evolves through time and the dynamical features of the time-variation within and between those nonconventional markets. To investigate such issues, a rolling windowing-based approach is employed to measure the long-range

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dependence temporal evolution for a large set composed of thirty alternative financial markets. In particular, the detrended fluctuation analysis (DFA) [14] is applied to each individual market return series by using a moving window in order to capture the dynamic self-similarity. Consequently, we will be able to dynamically determine whether the markets follow a pure random process or they are persistent or anti-persistent.

As DFA removes local trends and it is robust to non-stationarity, it is capable to detect the presence of long-range correlations while also avoid spurious detection of spurious correlations which are artifacts of non-stationarity [14]. The major concern of our work is to dynamically compute the Hurst exponent (HE) based on DFA over a rolling window. This way, we evaluate the level of long-range dependence in the underlying alternative (nonconventional) markets, through time. We use and analyze a long time period in order to obtain reliable dynamic HE estimates via the time-window based sampling. The presented approach is applied to a large set of alternative indices so as to produce robust statistical inference about the distribution of the estimated dynamic HEs, and statistically compare results within and between components of the markets. Eventually, the latter constitutes one more contribution to econophysics literature on this topical area.

The remainder of the paper is organized as follows. Section 2 describes the data and presents the complex DFA methodology. Section 3 presents the empirical results, whilst Section 4 discusses our findings and concludes.

## 2. Self-similarity modeling

The DFA [14] is an extremely efficient technique to capture self-similarity in nonstationary signals by measuring long-range dependence, i.e., temporal correlations. The algorithm for a given signal $y$ is given as follows:

a) Define the suite $x_N$ of the cumulative series of the original signal $y_i$ fluctuations about its mean:

$$x_N = \sum_{i=1}^{N} (y_i - \bar{y}) \tag{1}$$

b) Divide $x_N$ into boxes of equal length $n$.
c) In each box, fit the local trend of $x_N$ by a polynomial $P(n,N)$ that represents the local trend of the box. In our study, a polynomial of degree one is employed.
d) For the given $n$ box size, compute the root-mean-squared detrended fluctuation of the signal $x_N$ as:

$$F(n, N) = \sqrt{\frac{1}{N} \sum_{i=1}^{N} (x_1 - P(n, N))^2} \tag{2}$$

For each of the available $n$ box size, the last step is repeated to obtain the empirical relationship between the overall fluctuation $F(n, N)$ and the box size $n$:

$$F(n, N) \propto n^H \tag{3}$$

The Hurst exponent $H$ is estimated by running a regression of the $\log(F(n,N))$ on $\log(n)$. For $H = 0.5$, the dynamics of the original time series follow a random walk. For $0 < H < 0.5$, the series are anti-persistent. On the contrary, the series are persistent when $0.5 < H < 1$. At last, according to [14], when $H \geq 1$ autocorrelation exists but cease to be in a power-law form. In this study, a rolling (sliding) window of 300 observations is used to consistently and dynamically compute the DFA-based Hurst exponent ($H$). Hurst exponent is denoted as HE hereafter.

For each alternative market index, the Student $t$-test will be applied to check whether the mean of population of estimated dynamic Hurst exponents (HE) is equal to 0.5, greater than 0.5, or less than 0.5. Accordingly, the purpose of these tests is respectively to test whether the underlying original return time series follows a purely random, persistent, or anti-persistent dynamical pattern through time. In addition, the Brown-Foresyth $F$-test is employed to test the null hypothesis of ($i$) homogeneity of variance of estimated dynamic HEs within Islamic markets, ($ii$) homogeneity of variance of estimated dynamic HEs within sustainability, ecological, and ethical markets (non-Islamic), and ($iii$) the homogeneity of variance of the HEs across all markets. Indeed, the goal is to verify whether the distributions of the dynamic HEs are similar or different within and between each market.

## 3. Empirical findings

The dataset utilized in this paper comprises the daily closing prices $p(t)$ of thirty alternative (nonconventional) investment indices incorporated to the S&P Dow Jones from 29 February 2008 through 15 March 2018. The markets are distinguished into sixteen Islamic (Shariah) indices, nine sustainability indices, two ecological indices (Carbone and green), and three ethical indices. Our dataset is balanced between Islamic and non-Islamic alternative investment indices.

In particular, the alternative investment (market) indices considered in the current work are: S&P 500 Shariah Index, Dow Jones Islamic Market Titans 100 Index, S&P Global 1200 Shariah, Dow Jones Islamic Market World Index, Dow Jones Islamic Market Asia/Pacific Index, Dow Jones Islamic Market Europe Index, Dow Jones Islamic Market Turkey Index, Dow Jones Islamic Market Developed Markets Index, Dow Jones Islamic Market GCC Index (USD), Dow Jones Islamic Market Emerging ex-Frontier India Egypt and Peru Index, S&P BSE 500 SHARIAH, S&P BRIC Shariah Index, S&P Bangladesh BMI Shariah, S&P Global Infrastructure Shariah, S&P Global 1200 Shariah, S&P Global 1200 Shariah, S&P/BMV IPC Sustainable, Dow Jones Sustainability Nordic Index, Dow Jones Sustainability World Index, Dow Jones Sustainability Europe Index (EUR), Dow Jones Sustainability World Enlarged Index, Dow Jones Sustainability World Enlarged Index (EUR), Dow Jones Sustainability World Enlarged Index (USD), Dow Jones Sustainability World Developed Index, S&P BSE CARBONEX, S&P BSE GREENEX, DJSI Ethical Europe Low Volatility Index (EUR), DJSI Ethical Europe Low Volatility Index (USD), and S&P Ethical Pan Asia Select Dividend Opportunities Index.

The DFA is applied to the return series $r(t)$ computed as first-log differences of the price time series $p(t)$, namely $r(t) = \log(p(t)-\log(p(t-1))$ where $t$ is time script. The sample sliding-window is of length 300 days and moves forward in one day intervals. For illustration purposes, Fig. 1 displays some examples of DFA-based estimated dynamic Hurst exponents for the S&P 500 Shariah Index, Dow Jones Sustainability World Index, S&P BSE CARBONEX, S&P BSE GREENEX, DJSI Ethical Europe Low Volatility Index (USD), and S&P Ethical Pan Asia Select Dividend Opportunities Index. It is interesting to observe that the DFA-based estimated dynamic Hurst exponent for each one of these markets exhibits a cyclical behaviour; the period of consistent increase (November 2008 to December 2010, January 2013 to September 2014, August 2017 to January 2018) is followed by periods of consistent decrease (December 2010 to January 2013, September 2014 to August 2017). In fact, we discovered similar patterns for all remaining alternative market indices, not shown here for the sake of simplicity. In this regard, the returns of all alternative markets exhibit persistent or anti-persistent dynamics depending on the time cycle, which makes the amplitude of long-range dependence obviously time-varying. In Fig. 2 we exhibit the boxplots of DFA-based estimated dynamic Hurst exponents for each alternative market for the time period under study. A first look suggests that the distributions of DFA-based Hurst exponent estimates are not similar.