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Review

On the relay coupling of three fractional-order oscillators with time-delay consideration: Global and cluster synchronizations

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ABSTRACT

The relay coupling of three fractional-order two-stage oscillators in the presence of time delay has been explored theoretically, numerically and analogically. The global stabilization of the system in a finite time is proven through Hölder and Gronwall inequalities, as well as through inequality scaling skills. The Synchronization of the system is characterized in terms of its parameters (coupling strength and time delay) by using time series, two parameters phase diagrams and two parameters transverse Lyapunov exponent diagrams. It is found that for smaller delay values, the network exhibits global phase synchronization whereas for higher delay values, phase synchronization just occurs between the two indirectly connected units (cluster phase synchronization). Striking phenomena such as amplitudes' death and chaotic beats oscillations are also observed from this relay coupling of three fractional-order two-stage oscillators. Furthermore, PSpice simulation results of the analog electronic circuit are in perfect accordance with both theoretical and numerical results.

1. Introduction

Relay coupling has been paid much attention in the field of nonlinear dynamics [1–6]. Since its discovery, a primary interest in such studies has been focused on synchronization in its many different forms, from both theoretical and experimental point of view [4]. Applications on relay systems are increasingly found in various sectors. For instance, in real communication systems, one or many relay stations are set between emitter and receiver systems [6]. Similarly during a military parade, the brain serves as a relay between the members of a soldiers' group thus promoting the synchronization of their gestures. Work on relay coupled systems is becoming more and more important in the literature and the search for different types of synchronization in these relay coupled systems is well highlighted [1–5]. Many types of synchronization such as in phase, out-of-phase, complete and cluster synchronization as well as amplitudes death phenomenon have been reported in the last two decades in coupled systems considering time delays and conjugated variables [1–18]. Coupled map networks were used in Ref. [17] to assess the impact of heterogeneous delays on cluster synchronization. The authors found that heterogeneity in delays induces a rich cluster pattern as compared to homogeneous delays, while the parity of heterogeneous delay plays a crucial role in determining the mechanism of clusters' formation (driven or self-organized). Furthermore, in investigating the impact of a homogeneous delay on the phenomenon of phase synchronized clusters in coupled map networks [18], outcomes reveal that delay may lead to a completely different relation between dynamical and structural clusters, than that observed in the un-delayed case. In Ref. [7], based on the Lyapunov spectral as a function of time delay, the authors show the existence of the phase-flip bifurcation in synchronization of two excitable systems and two ecological systems when the state variables in the coupling are delayed.Thus, the coexistence of in-phase and out-of-phase are observed by both numerical and analogue simulations. The authors of Ref. [8] shown the existence of amplitudes death on the dynamic of two Chua circuits mutually coupled via conjugated variables. Their numer-

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ical and experimental results are in good agreement. These results have been reported in the relay coupled systems via dissimilar (conjugate) variables in the absence of time delay [4]. In Ref. [1], using two topologies of coupling: the full coupling and partial (relay) coupling, the authors investigated chaotic phase synchronization in three coupled chaotic oscillators. Their basin of attraction showed the coexistence between the complete (global) chaotic phase synchronization and partial (cluster) chaotic phase synchronization when the coupling strength and frequency are set as control parameters. They also showed that for the smaller values of coupling strength, the chaotic phase synchronization switching is induced by breakdown of complete chaotic phase synchronization. The issue of synchronization between two delay-coupled oscillators via a third element connected as a relay between the two delayed oscillators was addressed in Ref. [2]. The authors showed that synchronized dynamical states can happen over long distances through a relay regardless of the value of delay. In [3], the existence of generalized synchronization in relay coupled systems is demonstrated. Based on the Lyapunov spectrum of the full system, the authors identified conditions for the complete and generalized synchronization. On the other hand, considering the distance between phase-space neighbors, the authors used two nonlinear measures to amount the generalized synchronization in discretized time series. Sharma et *al.* [4] have shown that in the absence of time delay, relay coupling through conjugates variables has the same effect, namely: a transition from in-phase to out-of-phase oscillation as well as the transition to amplitude death and subsequently to the stabilization of a fixed point. Results of the research works mentioned in all the references cited previously are very interesting. However, there are aspects that merit more attention and which have not been considered. Firstly, the systems descriptions are restricted to the integer-order differentials equations, although it has already been often reported that many systems, e.g. in chemical and physical processes could rather be better or more accurately described by fractional-order differential equations [19–25]. Secondly, references on relay coupled systems above have not considered time delay in their approach; this limitation is a drawback in practical telecommunications. Thirdly, to the best of our knowledge, even finite-time synchronization has neither received enough attention in those relay coupled systems, nor analogical model of relay coupled circuits taking time delay into account have been reported so far in the above works. Whereas Refs. [25,26] have shown that practical applications of fractional-order delayed oscillators are more robust than their integer-order counterparts. Furthermore, in chaos-based communication the fractional-order derivative and time delay can be used as additional keys to secure messages. Moreover fractional-order systems offer one additional advantage: in many synchronization experiments, it is observed that the convergence time is shorter than in the case of integer-order systems, an advantage during the recovery of messages [25,26]. On the importance of fractional-order time delayed oscillators in telecommunication, some results with respect to their synchronization have been reported this recent years [27–30]. The sliding mode synchronization [27] can be cited among others, as well as the projective cluster synchronization [28], the global projective synchronization [29], the finite-time Mittag–Leffler synchronization [30], all for the fractional-order coupled oscillators with time delay. In the light of these breaches, we have four objectives in this work : (i) to consider the relay coupling of three fractional-order delayed oscillators; (ii) to determine a sufficient condition ensuring the practical global finite-time synchronization in relay coupled fractional-order delayed oscillators; (iii) to determine the parameters regions (coupling strength and time delay) for which the relay coupling of three fractional-order delayed oscillators exhibit the global synchronization, phase synchronization and cluster synchronization; (iv) to carry out an analogical study of the relay coupled fractional-order systems in PSpice software to validate the theoretical and numerical analysis.

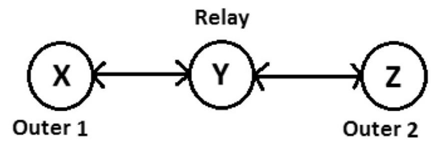

Fig. 1. Relay topology scheme in the present model.

The rest of the paper is as follows: In Section 2, the model description of relay systems and some preliminaries on the fractional-order derivatives theories are given. The main part of the finite-time stabilization in a relay coupling of three fractional-order systems in the presence of time delay is carried out in Section 3. Numerical simulations are given in Section 4. Section 5 deals with PSpice simulations to illustrate the effectiveness of the relay coupling of three fractional-order two-stage oscillators. Conclusion and remarks in Section 6 complete the paper.

2. Model description and preliminaries

2.1. Model description

Consider three fractional-order oscillators coupled as shown in Fig 1. Arrows between oscillators indicate bidirectional non adaptive coupling. In this topology, time delay between two oscillators directly coupled is taken into account. Each of the oscillators of Fig. 1 is a fractional-order two-stage oscillator. Fractional-order commensurate two-stage oscillator is well studied in [19,20].

Based on the mathematical model of fractional-order two-stage oscillator, as defined in [19,20], we can write the equations modeling the relay topology as follows: the outer 1 oscillator

$$\begin{cases} D^q X = AX + BF(X) + U_1(t,\tau) \\ X(t) = \alpha(t) \qquad t \in [-\tau, 0], \end{cases} \tag{1}$$

the outer 2 oscillator

$$\begin{cases} D^q Z = AZ + BF(Z) + U_2(t,\tau) \\ Z(t) = \varphi(t) \qquad t \in [-\tau, 0], \end{cases} \tag{2}$$

and the relay oscillator

$$\begin{cases} D^q Y = AY + BF(Y) + U_3(t,\tau) \\ Y(t) = \psi(t) \qquad t \in [-\tau, 0]. \end{cases} \tag{3}$$

D^q denotes the caputo derivative of order q. $A \in R^{4\times4}$ and $B \in R^{4\times4}$ are the linear matrix and $F \in R^4$ is a nonlinear vector all defined in [19,20]. The functions $U_1(t,\ \tau)$, $U_2(t,\ \tau)$ and $U_3(t,\ \tau)$ are the non adaptive feedback controllers integrating the time delay τ. The delay τ is the time it takes for the information to move from one oscillator to the next, with which it is directly coupled and conversely. $X = [x_1, x_2, x_3, x_4]^T$, $Z = [z_1, z_2, z_3, z_4]^T$ and $Y = [y_1, y_2, y_3, y_4]^T$, denote the outer 1, outer 2 and relay states respectively. According to Refs. [19,20], the matrix A and B can be written as follows: $A = \begin{bmatrix} 0 & 0 & 0 & \sigma_1 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & \sigma_2 \\ -1 & -1 & -1 & -\varepsilon \end{bmatrix}$, and $B = diag\begin{pmatrix} -\gamma_1 & 0 & -\gamma_2 & 0 \end{pmatrix}$, where σ_1, σ_2, γ_1, γ_2 and ε are the dimensionless parameters of the two-stage oscillators. The nonlinear vector F is defined such that for a state vector $X' = [x'_1, x'_2, x'_3, x'_4]^T$,

$$F\left(X'\right) = \left[exp(-x'_2 - x'_3) - 1, \quad 0, \quad exp(-x'_2) - 1, \quad 0\right]^T.$$

Using the above consideration, the control signals received by outer 1, outer 2 and relay oscillators are defined as follows: outer 1

$$U_1(t,\tau) = K_1(Y(t-\tau) - X), \tag{4}$$