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An evolutive discrete exchange economy model with heterogeneous preferences

Ahmad Naimzada[a], Marina Pireddu[b,*]

[a] Department of Economics, Management and Statistics, University of Milano-Bicocca, U6 Building, Piazza dell'Ateneo Nuovo 1, Milano 20126, Italy
[b] Department of Mathematics and its Applications, University of Milano-Bicocca, U5 Building, Via Cozzi 55, Milano 20125, Italy

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ABSTRACT

We propose an exchange economy evolutionary model with discrete time, in which there are two groups of agents characterized by different structures of preferences. The share updating mechanism depends in a monotone manner on the goods' consumption, described in terms of the calorie intakes. In such framework we investigate the existence of equilibria, their stability and the occurrence of multistability phenomena via a qualitative bifurcation analysis, which also highlights the presence of transcritical bifurcations.

1. Introduction

In the present paper we are going to show the richness of dynamic behaviors arising from the exchange economy evolutionary model introduced in [1] when considering time as discrete rather than continuous. Indeed, like in that paper we deal with an exchange economy setting in which agents are heterogeneous in the structure of preferences. Namely, the weights assigned to the two consumption goods in the Cobb–Douglas utility functions do not coincide across groups. Moreover, similarly to [1], the mechanism according to which shares are updated depends on goods' consumption, described in terms of the calorie intakes of the two groups of agents. However, instead of the linear, monotone dependence considered therein, we assume a nonlinear, but still monotone, relationship between the share updating mechanism and the (difference of the) calorie intakes. In particular, following [2,3], we do consider an exponential discrete replicator rule to describe the evolutionary mechanism. We recall that, according to [1], a monotone population growth rate is suitable to represent the long-run centuries-old trend, as the diet of a population group affects its long-term survival. More precisely, as observed in [4], biological payoff functions monotonically increasing in the calorie intake well describe food regimes characterized by a calorie shortage, and are thus appropriate to represent the long-run centuries-old trend before the industrial revolution.

The model we propose belongs to the line of research developed in [4–6] and inspired to the setting in [1]. In more detail, in [4,5] time is continuous and the focus is on the analysis of the local stability of the equilibria and on some of their static features, such as weak and strong coexistence between groups, assuming that endowments are respectively homogeneous and heterogeneous between groups. In those papers we replaced the monotone population growth rate assumed in [1] with a bell-shaped map, increasing with the calorie intake up to a certain threshold value, above which it becomes decreasing. Bell-shaped maps are indeed well-suited to describe the framework of contemporary developed countries and the negative effects of overconsumption on health and survival (see [5,7]). In [6] the evolutionary mechanism is based on the relative utility values realized by the two kinds of agents, rather than on biological payoffs.

As mentioned above, differently from [1], in the present paper we do consider time as discrete, rather than continuous. However, we stress that we do not deal with a numerical discretization of the model proposed therein. Indeed, our discretization is built on the assumptions we make on the evolutionary mechanism. In particular, we describe it via an exponential replicator rule (see [2,3]). The choice of dealing with a discrete-time, rather than continuous-time, model comes from the consideration that the former framework is more suitable to represent the sequence of actions which lead to the formation of the population shares. Namely, in view of

* Corresponding author.
E-mail addresses: ahmad.naimzada@unimib.it (A. Naimzada), marina.pireddu@unimib.it (M. Pireddu).

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embracing a new preference structure, agents need time to evaluate the satisfaction degree resulting from their previous choice, to gather information on the other lifestyles and to compare the various satisfaction levels, in order to make their next choice.

We find that the passage from continuous to discrete time is not innocuous in terms of results. More precisely, like in [1], in addition to the two trivial market stationary equilibria, in which just one of the two groups of agents is present, we find at most one nontrivial equilibrium, characterized by the coexistence between the two groups. We perform a qualitative bifurcation analysis on varying mainly the parameter measuring the heterogeneity in the structure of preferences between groups and we prove that the nontrivial equilibrium may emerge via a transcritical bifurcation. Thanks to our nonlinearity, such internal equilibrium can be stable or it can lose stability via a flip bifurcation, leading to the presence of oscillatory and chaotic orbits, while in [1] it is always stable. We stress that when it is locally stable, it may be surrounded by periodic or chaotic attractors, due to the presence of multistability phenomena.

The remainder of the paper is organized as follows. In Section 2 we present our model and we analyze the existence and local stability of the equilibria. In Section 3 we perform a qualitative bifurcation analysis, showing the possible dynamic behaviors for the system. In Section 4 we briefly discuss our results and describe possible future study directions.

2. The model

We start our discussion recalling the framework in [1], where the authors consider a continuous-time model describing an exchange economy with a continuum of agents, which may be of type α or of type β. There are two consumption goods, x and y, and agent preferences are described by Cobb-Douglas utility functions, i.e., $U_i(x,y)=x^i y^{1-i}$, for $i\in\{\alpha,\ \beta\}$, with $0<\beta<\alpha<1$. The quantity of good x (y) consumed by an agent of type $i\in\{\alpha,\ \beta\}$ is denoted by x_i (y_i). Both kinds of agents have the same endowments of the two goods, denoted respectively by w_x and w_y. The analysis is performed in terms of the relative price $p(t)=p_y(t)/p_x(t)$, where $p_x(t)$ and $p_y(t)$ are the prices at time t for goods x and y, respectively. The size of the population of kind α (β) at time t is denoted by $A(t)$ ($B(t)$) and the normalized variable $a(t)=A(t)/(A(t)+B(t))$ represents the population fraction composed by the agents of type α. For simplicity, we assume that the population size is normalized to 1, so that the fraction composed by the agents of type β is given by $1-a(t)$.

We now present the definition of market equilibrium, we will refer to in the remainder of the paper. With this respect, we stress that the only difference between the framework we are going to consider and the one in [1] is that, in order to take into account the complexity of the evolutive process of share formation and the time it requires, we assume that in our model time is discrete rather than continuous.

Definition 2.1. Given the economy and the population share $a(t)$, a market equilibrium at time t is a vector $(p^*(t), x_i^*(t), y_i^*(t))$, with $i\in\{\alpha,\ \beta\}$, such that:

-every kind of agent chooses a utility-maximizing consumption bundle, given $p^*(t)$;
-the markets for the two goods clear.

Simple computations show that, solving the consumer maximization problems for agents of type α and β and using a market clearing condition, the market equilibrium price is given by

$$p^*(t)=\frac{\left[1-\left(a(t)\alpha+(1-a(t))\beta\right)\right]w_x}{\left(a(t)\alpha+(1-a(t))\beta\right)w_y} \tag{2.1}$$

and the consumer equilibrium quantities of the two goods for an agent of type $i\in\{\alpha,\ \beta\}$ are

$$\begin{aligned} x_i^*(t)&=i(w_x+p^*(t)w_y)=\tfrac{iw_x}{a(t)\alpha+(1-a(t))\beta},\\ y_i^*(t)&=(1-i)\left(\tfrac{w_x}{p^*(t)}+w_y\right)=\tfrac{(1-i)w_y}{1-(a(t)\alpha+(1-a(t))\beta)}. \end{aligned} \tag{2.2}$$

See [1,5] for further mathematical details.

Once we specify a dynamical rule for the population share evolution, it is also possible to give the definition of market stationary equilibrium as follows.

Definition 2.2. Given the economy, the vector (a^*, p^*, x_i^*, y_i^*), $i\in\{\alpha,\beta\}$, is a market stationary equilibrium if a^* is constant and if, given a^*, (p^*, x_i^*, y_i^*), $i\in\{\alpha,\ \beta\}$, is a market equilibrium for every t.

We stress that, in the definition above, a^* is a stationary value in a dynamical, rather than in a general equilibrium, sense. Vice versa, for (p^*, x_i^*, y_i^*), $i\in\{\alpha,\beta\}$, in agreement with Definition 2.1, we say that it is an equilibrium if, given p^*, every kind of agent chooses a utility-maximizing consumption bundle and in the markets for the two goods demand equals supply. Moreover, in the market stationary equilibria, p^*, x_i^* and y_i^* have to remain constant over time. We remark that, in order to underline the different nature of a and of $(p,\ x_i,\ y_i), i\in\{\alpha,\ \beta\}$, in Definition 2.2 it would be more appropriate to denote the first component of a market stationary equilibrium using a different symbol from that employed in Definition 2.1 to denote market equilibria. However, not to overburden notation, in agreement with [1] we chose to denote all components of market stationary equilibria by a star. In fact, for the sake of brevity, we will identify market stationary equilibria just with the population share a, since it determines all other equilibrium components.

The market stationary equilibria, at which for every t the population shares, and thus also the market equilibrium price and the consumer equilibrium quantities, are constant, will be called trivial if they are not characterized by the coexistence between the two groups of agents, and nontrivial otherwise.

Let us now recall the dynamical rule for the population share evolution considered in [1] and based on a biological payoff.

The calorie intake $K_i(t)$ of an agent of type $i\in\{\alpha,\ \beta\}$ at time t is given by a linear combination of the units $x_i(t)$ and $y_i(t)$ of goods x and y he consumes, weighted respectively with the calories that each agent derives from the consumption of a unit of good x and of good y, i.e., $K_i(t)=c_x x_i(t)+c_y y_i(t)$. Denoting by $\overline{K}$ the calorie subsistence level, in [1] the growth rate of the population of type i is then assumed to be

$$K_i(t)-\overline{K},$$

so that the evolution of the two groups of consumers is described by the following system

$$\begin{cases} \frac{dA(t)}{dt}=(K_\alpha(t)-\overline{K})A(t)\\ \frac{dB(t)}{dt}=(K_\beta(t)-\overline{K})B(t) \end{cases}$$

or equivalently, in terms of the normalized variable $a(t)$, by

$$\frac{da(t)}{dt}=a(t)(1-a(t))(K_\alpha(t)-K_\beta(t)). \tag{2.3}$$

Hence, since for $i\in\{\alpha,\ \beta\}$

$$K_i(t)=\frac{i\,c_x w_x}{a(t)\alpha+(1-a(t))\beta}+\frac{(1-i)c_y w_y}{1-a(t)\alpha-(1-a(t))\beta}, \tag{2.4}$$

(2.3) can be rewritten as

$$\frac{da(t)}{dt}=(\alpha-\beta)a(t)(1-a(t))\times\left(\frac{c_x w_x}{a(t)\alpha+(1-a(t))\beta}-\frac{c_y w_y}{1-a(t)\alpha-(1-a(t))\beta}\right).$$