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An analysis of an impulsive stage structured integrated pest management model with refuge effect

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ABSTRACT

Integrated pest management (IPM) utilizes a combination of control methods in order to control pest populations in agricultural systems. Here, we construct a stage structured impulsive integrated pest management with added prey refuge. By considering the ability of pests to hide from management strategies, we establish properties for pest eradication and permanence of the proposed system. Simulations of eradicated and permanent solutions are also included in order to illustrate the behavior of pest and predator populations.

1. Introduction

Pests pose a variety of risks to their environments, from health issues to property damage. Farmers have constantly struggled with pest management and have tried numerous control methods to protect their crops. Over the past few decades, the struggle to control these pests has increased as they have developed resistance to classical control techniques. Unfortunately, this makes the task of pest eradication very difficult from both biological and economical standpoints. In recent years, finding more effective and efficient control methods for pest management has become necessary for crop management.

Crop managers have used a variety of techniques including but not limited to spraying of pesticides, manipulating natural enemies, and introducing parasites or disease in order to aid in pest management. In the 1970s and 1980s, the combination of control methods known as integrated pest management (IPM) became widely used [1]. The goal of IPM is to maintain pests at a tolerable level using a combination of biological, chemical, and cultural control. By maintaining the pest population at a specified economical threshold (ET), this allows for long-term management and becomes a more cost effective solution for pest control. Additionally, the implementation of multiple pest control strategies have been proven successful theoretically and experimentally [1].

The combination of these methods is used to reduce pest populations to tolerable levels while being less costly and having minimal effects on the environment. Biological control can be the use of natural enemies or introduction of pest-specific disease in order to eradicate pests or reduce pests to tolerable levels. The augmentation, or periodic release, of natural predators is like using a "living insecticide"; this is an artificial introduction of the predator used when pests have reached or exceeded the economic threshold (ET) [1,8]. Through mass production and genetic enhancement of the natural enemies, augmentation leads to an increase in control effectiveness. The addition of parasites and pest-specific disease are other biological control techniques used to manage pest populations. Chemical control is the use of pesticides that are sprayed on the crops in attempt to eradicate the pests. This can become costly over time and cannot ensure the complete eradication of pests. Persistent spraying of chemicals on pests results in the pests' eventual resistance to these chemical sprays. Also, the chemicals used have potential health risks to humans and crops [1].The modification of agroecosystems is known as cultural control. This is less commonly practiced due to factors such as climate and environmental conditions that cannot be controlled; a shift in these factors can be devastating to the crops if the agroecosystem has been manipulated. However, in order to ensure cost effective and long-term solutions, a combination of these methods are necessary.

A wide range of approaches have been taken in the attempt to create functional integrated pest management models. With this study, we aim to describe some of the previously used methods for IPM modeling and discuss shortcomings of these proposed

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models. By implementing means of biological control through the interaction of predators and prey, these models can be used in order to determine appropriate conditions needed for total eradication of the pests or maintaining the pests at the acceptable ET. The earliest models specifically focus on biological control by studying predator-prey interactions, which is the basis of IPM models. Analytic techniques can be applied to the proposed models to determine conditions under which pest eradication solutions are globally asymptotically stable or permanent solutions exist.

Because these pests commonly have complex life stages, considering stage structure in IPM models is necessary. Models that do not include stage structure assume that all age classes have the same density dependent rates, as well as the ability to reproduce and compete [9]. Due to differences in fitness and maturation of juvenile versus adult individuals, adding stage structure will help eliminate such assumptions. In recent years, the use of impulsive differential equations to model these systems has been proposed [2,6,9]. Because farmers cannot spray pesticides and release predators continuously, we model these activities using impulsive controls [1,2,10]. By including impulsive effects, we assume that spraying and releasing is performed at specified intervals in time. In addition, these pulses can be used to represent seasonal births or regular pulse behavior in births [9].

We first recall Song and Xiang's model that includes stage structure of the predator class and impulsive differential equations representing integrated pest management [1]. The proposed model is a two-prey one-predator system:

$$\begin{cases} \left.\begin{aligned} &\dot{x}_1(t) = b_1x_1(t)(1-x_1(t)) - \alpha x_1(t)x_2(t) - r_1x_1(t)y_2(t),\\ &\dot{x}_2(t) = b_2x_2(t)(1-x_2(t)) - \beta x_1(t)x_2(t) - r_2x_2(t)y_2(t),\\ &\dot{y}_1(t) = \tfrac{\lambda_1 r_1 x_1(t)y_2(t)}{1+r_1h_1x_1(t)} + \tfrac{\lambda_2 r_2 x_2(t)y_2(t)}{1+r_2h_2x_2(t)} - (m+\mu)y_1(t),\\ &\dot{y}_2(t) = my_1(t) - \mu y_2(t), \end{aligned}\right\} t \neq nT \\ \left.\begin{aligned} &\Delta x_1(t) = -E_1x_1(t),\\ &\Delta x_2(t) = -E_2x_2(t),\\ &\Delta y_1(t) = p_1,\\ &\Delta y_2(t) = p_2, \end{aligned}\right\}, t = nT. \end{cases} \tag{1}$$

For $t \neq nT$, we have our ordinary differential equation model composed of the two-prey, $x_i(i=1,2)$, and stage structured predator, $y_i(i=1,2)$, classes. In this model it is assumed that the mature predator class can feed on the two prey classes at a pest-specific predation rate $r_i(i=1,2)$, while immature predators are too weak. In turn, the handling time h and conversion rate λ represent the conversion of feeding to production of new predators. Additionally, the model includes the intrinsic growth rates $b_i(i=1,2)$, competitive prey interaction effects $\alpha>0$, $\beta>0$, and the maturation m and death μ rates of predators. At intervals of length T, the impulsive behavior is applied, where a pesticide spray kills a fraction of the prey, $E_i(i=1,2)$, and a number $p_1>0$, $p_2>0$ of immature and mature predators are released, respectively.

Song and Xiang established stability conditions for model (1) [1]. Specifically, the authors found conditions under which the eradicated pests solution are globally asymptotically stable. Conditions for system permanence were also established; these conditions can give information about how to appropriately maintain tolerable pest populations. By establishing these conditions, the authors have proposed an IPM model that can be useful. However, one limitation of this model is the continuous birth of new pests, a more realistic assumption is that births happen seasonally or have pulse-like behavior. Although authors previously considered periodic impulsive models such as those in [5] and [6], this model is the first to propose impulsive effects for an IPM model and outlines important results for other models to be discussed including the model proposed in this work.

Recently, Akman et al. [2] proposed a similar model and conducted stability analysis and added stochastic birth pulses. This model stems from system (1) and includes the impulsive behavior. Additionally, this model only considers one pest, but the pest population has stage structure to accommodate for the differences in fitness of juvenile and adult individuals. The authors first proposed the following stage structured, impulsive deterministic model:

$$\begin{cases} \left.\begin{aligned} &\dot{x}_1(t) = -m_xx_1(t) - rx_1y_2(t)\\ &\dot{x}_2(t) = m_xx_1(t) - rx_2y_2(t)\\ &\dot{y}_1(t) = \tfrac{\lambda r(x_1(t)+x_2(t))y_2(t)}{1+rh(x_1(t)+x_2(t))} - (m_y+\mu)y_1(t)\\ &\dot{y}_2(t) = my_1(t) - \mu y_2(t), \end{aligned}\right\}, t \neq nT \\ \left.\begin{aligned} &x_1(t^+) = (x_1(t) + b\exp(-(x_1(t)+x_2(t)))x_2(t))(1-E),\\ &x_2(t^+) = x_2(t)(1-E)\\ &y_1(t^+) = y_1(t) + p_1\\ &y_2(t^+) = y_2(t) + p_2 \end{aligned}\right\}, t = nT \end{cases} \tag{2}$$

The system consists of one prey x_i, $(i=1,2)$ and one predator $y_i(i=1,2)$ with two stage classes, juveniles and adults. The structure of this system is similar to (1), where it is assumed only adult predators feed. Additionally, the description of parameters align with Song and Xiang's model. The birth pulse follows a Ricker-type function $b\exp(-(x_1(t)+x_2(t))$ [9], where b is the intrinsic growth rate.

Similar to Song and Xiang, the authors established conditions for eradication and permanence for the one-prey one-predator model introduced. After conducting a stability analysis of the deterministic system (2), the authors added stochastic birth effects. Adding such stochastic events is an eventual goal of this work.

One limitation of the previously discussed models is the lack of a refuge effect, which is the focus of this work. The *refuge effect* is the ability of prey to hide from any outside interactions, including predation by natural enemies and spraying of pesticides. The ability to hide from predators and pesticides can lead to an increased pest population by decreasing the number of susceptible individuals in the population. In recent years, several authors have considered adding a refuge effect to predator-prey systems [7,11]. In order to understand the influence of prey refuge and the necessity of adding this component to previously IPM models, we discuss previous results.

Refuge use by prey considered in current models is either modeled by a constant number or proportion of the prey population being protected [7,11]. Authors have found that by adding a refuge effect, changes in the stability properties of the system explained by the addition of refuge occur [7]. A second order differential equation model was proposed in [7], in which the refuge is considered as a constant proportion of prey. The identification and conditions of stability were found for this predator-prey model. Additionally, stochastic effects of prey refuge have been considered in previous predator-prey models [11]. The refuge models in [7] and [11] only consider predator-prey interactions and not IPM.

In order to determine how the hiding prey influence the vulnerable prey population size and overall system dynamics, we propose adding a refuge effect to an IPM model. Conditions for eradication and permanence are stated later in this paper to determine how the prey refuge influence the stability of the system. Based on the previous models discussed here, we develop an IPM model with added pest refuge.

Aside from pest management, IPM models can potentially be extended to immunology. Instead of trying to completely rid of cancerous cells by radiation or chemotherapy, the goal of immunotherapy is to target and reduce cancer levels to thresholds in which they will eventually become latent or eradicated. Immunotherapy is like integrated pest management since it utilizes a variety of methods while being less costly than other harsh meth-