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Phase diagram and dynamics of dark-bright vector solitons in spin-orbit-coupled Bose–Einstein condensate

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ABSTRACT

We investigate the dynamics of dark-bright vector soliton solutions in a spin-orbit coupled Bose–Einstein condensate with repulsive interaction by the imaginary time evolution. The phase diagram is obtained numerically in spin-orbit coupled Bose–Einstein condensate. We find that the spin-orbit coupling favors miscibility, and the energy detuning between the Raman beam and atom dominates the separation phase of the Bose gases. We also find that the spin-orbit coupled strength affects interaction types (attractive or repulsive) between the two dark-bright solitons.

1. Introduction

Spin-orbit coupling (SOC) is an interaction between spin with motion of particle, which plays a special role in many areas. The SOC for electrons in condensed matter system is particularly important for many condensed matter fields, such as spin Hall effect, topological insulator, spintronic devices, etc [1–3]. The experimental realization of synthetic SOC by the two photon Raman process in ultracold atom gases has offered an ideal platform to study novel properties of Bose–Einstein condenses (BECs) [4–6]. Meanwhile, the SOC effect in Bose system is the key factor for half vortices [7], monopoles [8], domain walls [9–11], bright solitons [12,13], dark solitons [14], gap solitons [15], and skyrmions [16] etc.

Matter-wave vector solitons as the nonlinear state have attracted much attention in BECs with the tunable intra- and inter-component repulsive(attractive) atomic interactions, which is expressed by the solution of coupled Schrödinger equation at the mean-field level. According to the type of short-range interaction, these matter wave solitons can be divided into two types, i.e., dark soliton and bright soliton. The former is characterized by a concave density and a non-trial phase jump across their density notch in repulsive interaction system, while a bright soliton with a convex density peak against a negligible background in attractive interaction system [17–19]. Compared with scalar BECs, the multi-component BECs enrich the ample novel nonlinear structures, such as bright-bright solitons [20–22], dark-dark solitons [23], dark-bright solitons, which can be thought of as symbiotic solitons [24–27]. Recently, the dynamical properties of dark-bright solitons are carefully investigated by the multiscale expansion method in a reduced SOC-BEC system. The results show that the oscillator frequency of dark-bright solitons in a harmonic trap is close to the regular oscillator frequency without the SOC [28]. Due to the potential application in many fields, dark-bright solitons had attracted intensive attentions [17].

In this paper, we investigate dark-bright solitons in a quasi one dimensional SOC-BECs with repulsive interaction. In particular, we are interested in the effects of the SOC strength and Raman detuning of the system on the densities distribution of different component wave functions and others fundamental characteristic of dark-bright solitons. It will be shown that the SOC strength and the detuning between the Raman beam and energy states of the atoms have the opposite effects on the width of dark-bright solitons and the height of bright soliton amplitude. The paper is organized as follows. In Section 2, we introduce the model equation for dark-bright solitons in a SOC Bose system. Section 3 is devoted to compute the different phase distribution via numerical method for different detunings and SOC strengths. It is aimed to give a di-

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rect insight of the different state densities in an interval of parameters. In Section 4, the dark-bright solitons are calculated by numerical method for various parameter interval. In Section 5 we calculate the dynamic evolution of dark-bright soliton and interaction between two stationary dark-bright solitons for different SOC strengths. Section 6 is a brief summary.

2. Model equation

We consider a quasi-one-dimensional SOC-BEC described by the coupled nonlinear Schrödinger equations [12]:

$$i\hbar\frac{\partial}{\partial t}\phi = \left[\frac{1}{2m}(p_x - \hbar\alpha\sigma_y)^2 + \hbar\Omega\delta\sigma_z - g^{'}(\phi^\dagger\cdot\phi)\right]\phi, \quad (1)$$

where $\phi = (\phi_1, \phi_2)^T$, $\phi^\dagger\cdot\phi = |\phi_1|^2 + |\phi_2|^2$, ϕ_1 and ϕ_2 are the two component wave functions, m is the atom mass, σ is Pauli matrix, α represents the strength of SOC, Ω is the coupling strength between Raman beams with atom levels, $g^{'}$ is attractive atomic interactions. To find dark-bright solitons numerically, we start from the dimensionless coupled Schrödinger equations,

$$\begin{aligned} i\frac{\partial}{\partial t}\phi_D = & \left[-\frac{1}{2}\frac{\partial^2}{\partial x^2} + \delta + g(|\phi_D|^2 + |\phi_B|^2)\right]\phi_D + \gamma\frac{\partial}{\partial x}\phi_B, \\ i\frac{\partial}{\partial t}\phi_B = & \left[-\frac{1}{2}\frac{\partial^2}{\partial x^2} - \delta + g(|\phi_D|^2 + |\phi_B|^2)\right]\phi_B - \gamma\frac{\partial}{\partial x}\phi_D. \end{aligned} \quad (2)$$

where ϕ_D and ϕ_B are the dark or bright soliton solution, respectively, δ is the energy detuning between the Raman beam and the atoms, g is the interaction strength between atoms in the unit of $Ng^{'}\sqrt{m/(\hbar\Omega)}/\hbar$, γ represents the SOC strength in the unit of $\alpha\sqrt{\hbar\Omega/m}$. The sign of γ, δ and g are positive for the whole paper. The scaling energy unit is $\hbar\Omega$, length is $\sqrt{\hbar/(m\Omega)}$ and time is $1/\Omega$. For the sake of simplicity the intra and inter interaction are set to be equal to g [14]. The normalization wave function is determined via $\int_{-\infty}^{\infty}(|\phi_D|^2 + |\phi_B|^2)dx = N$. N is in direct proportion to the total atom numbers. The dark-bright solitons in the SOC-BECs can be experimentally achieved by the counterflow of two hyperfine states of ^{87}Rb. The physical parameters can be similar as those in the experiment. The SOC γ depends on the laser wave length and the relative angle between the two Raman beams; while the energy difference δ can be easily tuned by changing the relative frequency of the two lasers. The units of the length and time are 1.7 µm and 4 ms, respectively, the total atom number is $N = 5 * 10^4$ level [4,27]. Eq. (2) are the starting point of the numerical calculations throughout this paper.

By solving Eq. (2) numerically, one is able to obtain the dark-bright solitons solution. To solve Eq. (2), we can first differentiate it by using split-time-step Crank–Nicolson method along with the homogeneous Neumann boundary condition, and then evaluate several thousands of steps in imaginary time until the invariable energy is reached. Every soliton(dark or bright) solution of Eq. (2) lives on a background which may be rest or highly oscillatory. Different from bright solitons cases, asymptotic behavior of dark soliton on the two sides of boundary is two different cases, that is, $\phi_D \to A_\pm e^{ikx}$, where A_+ and A_- are not the same complex constants and k is a real constant. According to the value of k, the background of soliton is rest ($k = 0$) or oscillatory ($k \neq 0$). It is not an easy task to find a stable dark-bright soliton in the case of highly oscillatory background, so we set the rest background case for simplicity [29].

3. Phase diagram in SOC Bose gas mixtures

In this section, we study the dark-bright soliton solutions in the SOC Bose gas mixtures. The dark-bright solitons have been created experimentally by filling the dark state with different hyperfine states of ^{87}Rb atoms. The phase of dark soliton can be imprinted and transferred bright soliton state atoms into dark one by the two-photon resonance process simultaneously [26]. When the SOC effect is presented in the Bose system, more ample phenomena occur. Here we carefully checked the different solutions of Eq. (2) in diverse parameter regions. The numerical result shows there exist three kinds of soliton solutions, i.e., dark soliton states, dark-bright solitons and stripe solitons, as shown in Fig. 1. We choose the wavefuction as the order of these phase transitions and use the properties of bright soliton solutions of Eq. (2) to distinguish the three different states of the system. When the amplitude of bright soliton of the dark-bright soliton solutions is less than certain value (e.g. 1% of the amplitude of dark soliton background), which is almost invisible, the state is defined as a single dark soliton. When the amplitude of bright soliton increases with the increasing SOC constant γ and detuning δ until the second peaks of the bright soliton of vector dark-bright solitons emerge, we call it as dark-bright soliton state. Otherwise, we define the states as stripe solitons, which have been intensively investigated in many literatures. Here we just focus the properties of vector dark-bright solitons in the SOC-BECs.

In fact, the dark and dark-bright soliton states form the separation phase and stripe soliton is a mixture phase of SOC Bose gases. From Fig. 1 we observe that $|\phi_{D,\ B}|^2$ evolves from one dark-bright soliton distribution to many nodes one when the SOC constant γ is increased, which are typical strip solitons in 1D system [30–32] and are mixture states of different hyperfine states. Particularly the mixture states are different from the same component strip solitons, which are total bright or dark soliton [13,14,33]. However, we can find that the emergence condition of the stripe solitons in Fig. 1 is the same to the case of region II of Fig. 1 in Ref.[13] when regarding to the linear dispersion relation of Eq. (2). At this time, the lowest energy of the stripe solitons may have either a positive or negative momentum in the linear dispersion relation or they can be a linear superposition of both modes, thus forming the stripe phase. With the increasing of the detuning δ, the components $|\phi_{D,\ B}|^2$ take a transformation from stripe soliton to one dark soliton state or dark-bright soliton, as shown in Fig. 1. This result shows that the SOC favors miscibility and the detuning between the Raman beam and atom energy levels dominates separation phase of the Bose gases. It should be pointed out that the presented phase diagram is different from the early result which mainly concentrate the different inter and intra interaction effect on the half-quantum vortex condensate or spin-spiral condensate in two dimension [34]. And it is not the same to another important result which focus on spin polarization and momentum as a function of Raman coupling constant Ω and critical density [35]. The stripe phase locates at the bottom of phase diagram and the separation phase lies on the top of Fig. 1, which is consisted with the recently experimental result [36].

As the SOC γ and detuning δ are set to zero, Eq. (2) reduces to the integrable Manakov limit when the intra and inter interactions are equal to the same constant [37]. Kivshar et al. has studied the dark-dark soliton solutions of coupled nonlinear equation in the normal group-velocity dispersion region, which is equal to repulsive interaction between atoms in BEC system[38]. A general dark soliton solution may have different intensities for two polarization modes. At this time the solution can be written in the form

$$\begin{aligned} \phi_{D_1} &= U_0(\cos\theta_1\tanh Z + i\sin\theta_1)\exp(i\Theta_1), \\ \phi_{D_2} &= V_0(\cos\theta_2\tanh Z + i\sin\theta_2)\exp(i\Theta_2). \end{aligned} \quad (3)$$

where $Z = U_0\cos\theta_1(x + t/W - \tau_0)$, $\Theta_1 = k_1x + (U_0^2 + V_0^2 + k_1^2)t$, $\Theta_2 = k_2x + (U_0^2 + V_0^2 + k_2^2)t$ and the parameters U_0, V_0, W, k_1 and k_2 are coupled by the following relations: $U_0\cos\theta_1 = V_0\cos\theta_2$, $W^{-1} = U_0\sin(\theta_1 + \theta_2)/\cos\theta_2 + (k_1 + k_2)$, $k_2 - k_1 = \sin(\theta_1 - \theta_2)/\cos\theta_2$. Here, ϕ_{D_1} and ϕ_{D_2} are the two component of dark solitons solu-