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Synchronization of time delay systems with non-diagonal complex scaling functions

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ABSTRACT

This paper deals with a kind of synchronization of dynamical systems with complex variables which is called generalized complex modified hybrid function projective synchronization (GCMHFPS) of time delay complex chaotic (hyperchaotic) systems. In other words, that the time delay complex systems can be synchronized up to a complex function transformation matrix. Moreover, the elements of the transformation matrix are complex functions of the states of drive system and time, where this matrix is not square. The idea of an active control method based on complex Krasovskii–Lyapunov functional is used to achieve GCMHFPS of time delay complex systems. Furthermore, based on the non-diagonal complex function transformation matrix, the modules and phases errors between one state of the complex response system and more than one of the states of the drive system are studied which have not been discussed before as far as we know. The analytical expression regarding the stability of this technique is derived and excellent agreement is found upon comparison with numerical calculations. In particular, we show through studying the time evolution of error, modulus and phase that the proposed scheme is effective for controlling time delay complex systems.

1. Introduction

In many realistic systems models such as biological, chemical, economical, and medical systems, a time delay appears due to many reasons. Its appearance is usually prompted by a physical nature of a plant or being introduced artificially to model a sampling effect [1–3]. Due to the new emerging applications in engineering, biology and other fields combined with new theoretical results, time delay systems have been actually the subject of intensive research works. Of course applications motivate the need of a theory, which in return makes the control applications possible. The presence of delays in these systems are often a source of instability and poor performance, and greatly increase the difficulty of studying stability and control. Stability of systems with time delay has been studied for decades and many results have been reported, see, e.g. [4–7], and references therein.

In 1990, Pecora and Carroll [8] investigated the synchronization of two systems connected with common signals and gave a criterion of the Lyapunov exponents. The common signals are as constraints between the two systems. Based on this idea, the synchronized circuits for chaos were developed by Carroll and Pecora [9]. Since then, one focused on developing control methods and schemes to achieve the different types of synchronization of chaotic and hyperchaotic systems [10–15]. In the last twenty years, many of mathematicians, engineers and biological scientists turned their attention on synchronization for complex nonlinear systems, because of its superabundant potential applications in chemical oscillations, detuned laser, secure communications, rotating fluids and particle beam in high energy accelerators. Different types of synchronization phenomena have been found in a variety of complex nonlinear systems such as complete synchronization (CS) [16], projective synchronization (PS) [17], chaotic phase synchronization (CPS) [18], lag synchronization (LS) [19], combination-combination synchronization (CCS) [20], generalized function projective synchronization (GFPS) [21], hybrid modified function projective synchronization (HMFPS) [22], and so on.

Obviously, for the above synchronization types the elements of the scaling matrix are real numbers real functions. In fact, the scaling matrix can be complex for complex dynamical systems. So, recently some new kinds of synchronization of complex dynamical systems are introduced. These new kinds have been studied

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for several examples of both chaotic and hyperchaotic complex nonlinear systems [23–26]. Zhang et al. [23] discussed modified projective synchronization with complex scaling factors (MPS) of uncertain parameters. Mahmoud and Mahmoud [24] investigated complex MPS (CMPS) of two chaotic complex systems. Mahmoud [25] designed a scheme to achieve the complex complete synchronization (CCS) of two nonidentical hyperchaotic complex nonlinear systems based on Lyapunov functions. Liu and Zhang [26] presented complex function projective synchronization (CFPS) of complex chaotic systems and its applications in secure communication. Recently, Liu et al. [27–29] presented general schemes for complex synchronization. In [27], they introduced a type of modified generalized projective synchronization with complex transformation matrix (CMGPS) of fractional-order real and complex systems with the same dimension and different structures. While in [28], they studied adaptive modified projective synchronization with complex scaling matrix (ACMPS) for two n-dimensional complex chaotic (hyperchaotic) systems with uncertain complex parameters. Also they discussed the adaptive modified hybrid function projective synchronization with complex function transformation matrix (CMHFPS) of different dimensional chaotic (hyperchaotic) complex systems with unknown complex parameters [29].

On the other hand, in secure communication and encryption schemes, it is better to use chaotic and hyperchaotic time delay complex systems. Due to finite signal transmission times, switching speeds and memory effects with time delays are ubiquitous in nature. Furthermore, complex nonlinear systems can be used to transmit information and to improve security [17,26,30]. Delay systems thus can be an interesting topic in synchronization and so far many works have been reported about the synchronization problem of delayed chaotic systems for both theoretical analysis and practical applications such as complete synchronization [31], projective synchronization [32], modified function projective synchronization [33]. Most of these results deal with general types of synchronization of real systems (their states variables are real), while there are only a remarkably few numbers of papers in the literature about the synchronization of time delay complex systems. To the best of our knowledge, Zheng [34] studied the synchronization of a class of time delay complex chaotic systems with discontinuous coupling. In [35], the authors discussed hybrid projective synchronization of time delay fractional-order chaotic complex systems. Mahmoud et al. [36] studied the modified time delay complex Lü system. Moreover, the synchronization of the modified time delay complex system is studied using the active control. Han and Zhang [37] investigated the complex function projective synchronization for complex dynamical networks with coupling time delays.

The main contribution of this paper is to state a new type of complex synchronization which is the generalized complex modified hybrid function projective synchronization (GCMHFPS) for two different dimensional time delay complex systems. A scheme is designed to achieve GCMHFPS. It may be considered as a generalization of several types (more than 17 types) of synchronization in the literature. Also in this article we will consider time delay complex systems which can be used for some applications in engineering, e.g. secure communications, chemistry, neuroscience and biology. Regarding to previous works such as [24,25,37], the complex transformation matrix is taken as diagonal matrix, so one studied the modulus and phase between one complex state of the response and one complex state of the drive. However, in our paper we will study the modulus and phase between one complex state of the response system with more than one complex state of the drive system. Moreover, in our scheme we will not separate the complex variables to its real and imaginary parts. In other words, the error of the synchronization, the controller and Krasovskii–Lyapunov function will depend on complex variables.

This paper is organized as follows: in Section 2, we give the definition of the GCMHFPS and design the scheme to achieve it for two different dimension of complex time delay systems. This scheme is based on active control technique and the Lyapunov stability theory. Section 3 discusses the results of numerical simulation of the proposed scheme. Two examples are given, the first one is devoted to study the GCMHFPS between two different dimensions, time delay complex Lorenz system [39] and modified time delay complex Lü system [36]. The second example is discussed GCMHFPS for two identical modified time delay complex Lü. Finally, Section 4 draws some conclusions.

2. Problem formulation

In this section, we will state the scheme for GCMHFPS of two different dimension of complex time delay systems. The drive time delay complex system can be described as:

$$\begin{aligned}\dot{\boldsymbol{x}}(t) &= \boldsymbol{x}^r(t) + j\boldsymbol{x}^i(t) = \boldsymbol{A}_1\boldsymbol{x}(t) + \boldsymbol{B}_1\boldsymbol{x}(t-\tau) + \boldsymbol{F}_1(\boldsymbol{x}(t), \boldsymbol{x}(t-\tau)),\\ \boldsymbol{x}(t) &= \boldsymbol{x}_0(t), t \in [-\tau, 0],\end{aligned} \tag{1}$$

where $\boldsymbol{x} = (x_1, x_2, \dots, x_n)^T$ is the state complex vector of drive system, $x_k = x_k^r + jx_k^i, k = 1, \dots, n, j = \sqrt{-1}, \boldsymbol{A}_1$ and $\boldsymbol{B}_1 \in \mathbb{R}^{n\times n}$ are real (or complex) matrices of the system parameters, T denotes transpose, $\boldsymbol{F}_1 = (F_{11}, F_{12}, \dots, F_{1n})^T$ is a vector of nonlinear complex functions, $\tau \geq 0$ is a constant time delay and superscripts r and i stand for the real and imaginary parts of the state complex vector $\boldsymbol{x}$.

The controlled response system as time delay complex system is given by:

$$\begin{aligned}\dot{\boldsymbol{y}}(t) &= \boldsymbol{y}^r(t) + j\boldsymbol{y}^i(t)\\ &= \boldsymbol{A}_2\boldsymbol{y}(t) + \boldsymbol{B}_2\boldsymbol{y}(t-\sigma) + \boldsymbol{F}_2(\boldsymbol{y}(t), \boldsymbol{y}(t-\sigma)) + \boldsymbol{\Theta},\\ \boldsymbol{y}(t) &= \boldsymbol{y}_0(t), t \in [-\sigma, 0],\end{aligned} \tag{2}$$

where $\boldsymbol{y} = (y_1, y_2, \dots, y_m)^T$ is the state complex vector of the response system, $y_l = y_l^r + jy_l^i, l = 1, \dots, m, \boldsymbol{A}_2$ and $\boldsymbol{B}_2 \in \mathbb{R}^{m\times m}$ are real (or complex) matrices of the system parameters, $\sigma \geq 0$ is a constant time delay, $\boldsymbol{F}_2 = (F_{21}, F_{22}, \dots, F_{2m})^T$ is a vector of nonlinear complex functions and $\boldsymbol{\Theta} = \boldsymbol{\Theta}^r + j\boldsymbol{\Theta}^i = (\Theta_1, \Theta_2, \dots, \Theta_m)^T, \Theta_l = \Theta_l^r + j\Theta_l^i$ is the controller that needs to be designed.

Definition 1. The complex delay drive system (1) and the complex delay response system (2), can exhibit GCMHFPS with $\boldsymbol{\Psi}(\boldsymbol{x}(t),t)$, if there exists a complex controller $\boldsymbol{\Theta} \in \mathbb{C}^{m\times 1}$ such that

$$\lim_{t\to\infty} \|\boldsymbol{e}(t)\| = \lim_{t\to\infty} \|\boldsymbol{y}(t) - \boldsymbol{\Psi}(\boldsymbol{x}(t), t)\boldsymbol{x}(t)\| = 0, \tag{3}$$

where $\boldsymbol{\Psi}(\boldsymbol{x}(t),t) = [\psi_{lk}]_{m\times n} \in \mathbb{C}^{m\times n}$, $(l = 1, 2, \dots, m; k = 1, 2, \dots, n)$ is defined as a complex function transformation matrix of the drive system (1), $\psi_{lk} = \psi_{lk}^r + j\psi_{lk}^i$ and all of its elements should be bounded and differentiable.

Remark 1. Many chaotic communication schemes, such as GCMFPS, GCFPS, CMHFPS which depend on chaotic systems with low-dimensional and have only one positive Lyapunov exponent are not as secure as expected and can be successfully unmasked. To ameliorate the security, it has been proposed to use time delay systems which have infinite dimensional and multiple positive Lyapunov exponents. So our scheme which depends on time delay systems is better than the previous schemes such as GCMFPS, GCFPS, and CMHFPS. Beside the existence of time delay systems, the elements of the complex function transformation matrix can be encryption keys that be used to improve the security.

Remark 2. Several types of complex synchronization are special cases of GCMHFPS of time delay complex systems just if we put $\tau = \sigma = 0$. For example, the generalized complex function projective synchronization (GCFPS) [26], complex modified hybrid function projective synchronization CMHFPS [29], complex modified