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A spherical conformal contact model considering frictional and microscopic factors based on fractal theory

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ABSTRACT

Spherical conformal contact is widely used in engineering structures. The existing solution of the spherical conformal contact problem always ignores the microscopic characteristics or friction of the spherical surfaces. Therefore, the current paper presents a spherical fractal model to characterize the contact state of spherical pairs considering the microscopic topography of rough spherical surface and the factor of friction. Firstly, a method of characterizing the microscopic topography of rough spherical surface is proposed based on three-dimensional Weierstrass–Mandelbrot function. The fractal contact model of the single asperity is developed by Hertz theory in combination with elasticity. Then, the macroscopic parameters are introduced to construct the contact surface coefficient. The area distribution function under conformal contact region is obtained. Considering the friction factor of the conformal contact region, the microcontact model of the spherical conformal contact is developed based on fractal theory. Finally, the formula between (elastic, elastic-plastic, plastic) contact area, (elastic, elastic-plastic, plastic) contact load and the key parameters (fractal parameters and macro parameters) are derived based on the proposed model. The relationship between the actual contact area and the normal load of the contact region is established. Numerical results show that the proposed model is more accurate for the analysis of the spherical surface contact area and contact load.

1. Introduction

The classical fractal contact theory is always limited to the plane or cylindrical contact problems. It cannot be used to solve the spherical conformal contact which exists in many engineering structures such as air plane's landing gear, bridge support, sliding bearings etc. The main failure mode of the conformal contact between the inner and outer surfaces is the sliding wear between the two contact surfaces. The actual area and load of the contact pair are the important factors that affect the friction and wear. Therefore, it is necessary to study the actual area and load of spherical conformal contact. But there are two issues among the current literature when calculating the actual area and load of spherical conformal contact. One is ignoring the microscopic characteristics of the spherical surface; the other is that the factor of friction is always ignored. However, all natural surfaces and surfaces of engineering have the surface roughness at different length scales, extending from atomic dimensions to the linear size of the object of study. Surface roughness is one of the most important factor researchers should consider in many engineering applications.

Many researchers studied the spherical conformal contact problems. The problem of frictionless sphere in a conforming cavity has been analyzed by Goodman & Keer using numerical methods appropriate to spherical bodies [1]. Steuermann has found the pressure distribution by an infinite series of known function for profiles of planar symmetry and axi-symmetry [2], the findings of Steuermann are though better than that of Hertz by inclusion of higher terms in the description of the profile, but this model is still limited in the assumption of elastic half-space, and its error rises rapidly with the increase of contact arc angle or with the decrease of clearance between sphere and spherical cavity [3]. Steuermann model was then modified by Liu and used to deal with the contact pressure distribution. This model can only explain the normal force-displacement relationship in a small scope of clearance and deformation. The error is even greater than Hertz model when exceeding this scope [4]. Fang established a friction-free conformal contact model by combining with theoretical analysis and numerical methods, which can calculate the contact pressure distribution of conformal contact between a sphere and a spherical cavity [5]. According to the above analysis, the existing studies on the contact area and load of spherical conformal contact do not consider the factor of friction and the microscopic characteristics of the sur-

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face, such as the morphology of the contact surface and the effect of surface roughness on contact ability.

The elastic-plastic contact of a sphere and a flat is a fundamental problem in contact mechanics. Chang et al. developed the first elastic-plastic contact model for rough surfaces (Chang–Etsion–Bogy (CEB) model) [6]. Kogut and Etsion (KE model) developed a finite element (FE) model to analyze the elastic-plastic contact of a deformable spherical asperity and a rigid flat [7]. L. Li developed a model for the contact area and static friction of nominally flat rough surfaces and rough spherical surfaces [8]. A model for an elastic–plastic contact between a deformable sphere and a rigid flat under combined normal and tangential loading with full stick contact condition was developed by V. Brizmer [9].

For the sake of considering the effect of the microcosmic factor on calculating the contact area and load, the fractal theory is introduced. This theory was proposed by Mandelbort and successfully applied to the British coastline [10]. Many researchers studied its application in engineering. Iasef Md Rian explored the application of fractal geometry to architecture and art [11]. Yao Liu proposed a two-stage method for fractal dimension calculation of the mechanical equipment rough surface profile based on fractal theory [12]. Mohamad' study shows that fractal geometry is an effective method for the study of the corrosion mechanism of the surface [13]. Guoxiong Chen proposed a matched filtering method for separating magnetic anomalies [14]. The hydraulic characteristic of the rock fracture network was studied based on the fractal theory [15]. Jiang Shuyun predicted the thermal contact resistance of two contact surfaces [16].

There are many studies on the contact models based on fractal theory, including on plane and cylindrical surfaces. Majumdar and Bhushan established the MB fractal contact model [17]. In their model, the fractal theory was applied to the two rough surfaces contact analysis for the first time. Then many subsequent studies proved the validity of the MB model [18-20]. Wang and Komvopoulos established the WK fractal contact model by modifying the area distribution function and using the elastic-plastic contact analysis [21]. Yan and Komvopoulos used fractal theory to study the contact problem of the three-dimensional (3D) rough surface [22]. Morag Etsiont argued that the deformation of the asperity was transformed from the elastic deformation to the plastic deformation. The elastic-plastic correction model of rough surface based on the MB model was proposed, namely the ME model [23]. Qi studied the fractal model to calculate the normal contact stiffness for spheroidal contact bodies [24]. Kang theoretically investigated the contact area and the negative friction velocity slope on dynamic instability of spherical joint [25].

The above contact models based on fractal theory mainly concentrate on rough "plane" or cylindrical surfaces, while the studies on rough "conformal spherical" surfaces are limited; in addition, most of them ignore the friction factor.

Therefore, this paper is to provide a fractal model to characterize the contact of rough conformal spherical surfaces with the friction factor considered. The paper is structured as follows: after an introduction, we present the spherical fractal theory in Section 2. Then, the microcontact model of the spherical conformal contact based on spherical fractal theory is presented in Section 3. Moreover, we give the numerical results and discussions in Section 4. A conclusion is drawn in Section 5.

2. Spherical fractal theory

2.1. Fractal characterization of rough spherical surface

Weierstrass (1872) derived a famous function which is everywhere continuous but everywhere not differentiable. The real part of the function is frequently used to characterize fractal rough profile, called the Weierstrass–Mandelbrot function (WM function). The original WM function is

$$y(x) = G^{D-1}\sum_{n=n_l}^{\infty}\gamma^{-(2-D)n}\cos(2\pi\gamma^n x), \quad 1 < D\langle 2, \gamma\rangle 1 \tag{1}$$

where D is the fractal dimension of profile, G is the fractal roughness which is a height scale parameter independent of frequency, x is the profile's horizontal coordinate, γ^n is the spatial frequency of the random profile, n is a frequency index, n_l is the number corresponding to the lowest cut off frequency of the profile.

Let $x=\rho\theta$, and the fractal profile is superimposed on the circle curve, the original WM function can be transformed into the new equation.

$$\rho(\theta) = \rho + G^{D-1}\sum_{n=n_l}^{\infty}\gamma^{-(2-D)n}\cos(2\pi\gamma^n\rho\theta), \quad 0\le\theta\le 2\pi \tag{2}$$

where ρ is the polar radius and θ is the polar angle

Fig. 1(a) shows the fractal profile when $L=20$ mm, $G=10^{-8}$ mm and $D=1.3$ in Eq. (1), and Fig. 1(b) shows the fractal profile with the condition of $L=20$ mm, $G=10^{-8}$ mm and $D=1.6$ in Eq. (1). The new two-dimensional WM function, that is Eq. (2), is employed to describe the circular fractal profile with a nominal radius of ρ. The simulation of circular fractal profiles is shown in Fig. 1(c) and Fig. 1(d) after setting the radius $\rho=0.05$ mm, $G=10^{-8}$ mm, $D=1.3$ and $D=1.6$ respectively. The function expands the application scope of original WM function. Fig. 1 shows that the profile curve in Fig. 1(a) and (c) are rougher than that in Fig. 1(b) and (d), it indicates that the fluctuation of the profile curve is larger when the fractal dimension D is 1.3 than 1.6. The circular fractal profile tends to be smoother when the fractal dimension D increased from 1.3 to 1.6, which is similar to the results of the linear simulation in Fig. 1(a) and (b). It indicates the existence and validity of the circular fractal profile.

Ausloos and Berman generalized the height distribution function of asperities on the 3D rough surface to describe the three-dimensional stochastic process by introducing more parameters in the original WM function, that is the Ausloos–Berman (AB) function, W.Yan and K. Komvopoulos derived three-dimensional (3D) WM function which can describe the microscopic features of rough surface based on AB function as follows [22]

$$\begin{aligned} z(x,y) = L\left(\frac{G}{L}\right)^{D_s-2}\left(\frac{\ln\gamma}{M}\right)^{\frac{1}{2}}\sum_{m=1}^{M}\sum_{n=0}^{n_{\max}}\gamma^{(D_s-3)n} \\ \cdot\left\{\cos\phi_{m,n} - \cos\left[\frac{2\pi\gamma^n\sqrt{x^2+y^2}}{L}\cos\right.\right. \\ \left.\left.\times\left(\arctan\left(\frac{y}{x}\right)-\frac{\pi m}{M}\right)+\phi_{m,n}\right]\right\}, \end{aligned} \tag{3}$$

where D_S $(2\le D_S\le 3)$ is the surface fractal dimension, M represents quantity and intensity of peaks which are composed of the surface, λ_n is the reciprocal of the random profile's space frequency, $\gamma^n=1/\lambda_n$, $\alpha_m=\pi m/M$ represents the arbitrary angle, a random number generator is used to generate random phase ϕ_n in the interval $[0,2\pi]$, the parameter k is a wave number which is related to the sample size $k=2\pi/L$, where L is the sample length, the anisotropy of the surface geometry is controlled by the magnitude of A_m. $n_{\max}=\mathrm{int}[log(L/L_s)/log\gamma]$, L_s is the sampling length. The quantity and intensity of peaks increase with the increase of M, which makes the surface roughness larger, surfaces possess cylindrical corrugations with the condition of $M=1$, λ denotes the profile density, which is a constant greater than 1, it's appropriate for random surface that obeys the normal distribution with $\gamma=1.5$ (suitable for high spectral density and random phase), α_m is used to offset the ridges in the azimuthal direction.