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A nonlinear Timoshenko beam formulation based on the modified couple stress theory

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ABSTRACT

This paper presents a nonlinear size-dependent Timoshenko beam model based on the modified couple stress theory, a non-classical continuum theory capable of capturing the size effects. The nonlinear behavior of the new model is due to the present of induced mid-plane stretching, a prevalent phenomenon in beams with two immovable supports. The Hamilton principle is employed to determine the governing partial differential equations as well as the boundary conditions. A hinged–hinged beam is chosen as an example to delineate the nonlinear size-dependent static and free-vibration behaviors of the derived formulation. The solution for the static bending is obtained numerically. The solution for the free-vibration is presented analytically utilizing the method of multiple scales, one of the perturbation techniques.

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1. Introduction

Microbeams are widely used in micro- and nano-electromechanical systems (MEMS and NEMS) such as vibration shock sensors [1], electro-statically excited micro-actuators [2–4], micro-switches [5] and atomic force microscopes (AFM) [6]. The thickness of beams used in MEMS, NEMS is in the order of microns and sub-microns. The size-dependent static and vibration behaviors in micro-scales have experimentally been validated. For example in the micro-torsion test of thin copper wires, Fleck et al. [7] indicated that decrease of wires diameter results in a noteworthy enhancement of the torsional hardening. Stolken et al. [8] reported a notable increase of plastic work hardening caused by the decrease of beam thickness in the micro-bending test of thin nickel beams. Also, size-dependent behaviors have been detected in some kinds of polymers. For instance, during micro-bending tests of beams made of epoxy polymers, Lam et al. [9] observed a significant enhancement of bending rigidity caused by the beam thickness reduction. McFarland et al. [10] detected a considerable difference between the stiffness values predicted by the classical beam theory and the stiffness values obtained during a bending test of polypropylene micro-cantilever. These experiments reveal that the size-dependent behavior is an inherent property of materials which appears for a beam when the characteristic size such as thickness or diameter is close to the internal material length scale parameter [11].

The classical continuum mechanics theories are not capable of prediction and explanation of the size-dependent behaviors which occur in micron- and sub-micron-scale structures. However, some non-classical continuum theories such as higher-order gradient theories and the couple stress theory have been developed such that they are acceptably able to interpret the size-dependencies.

In 1960s some researchers such as Mindlin, Touplin and Koiter introduced the couple stress elasticity theory which proposes two higher-order material length scale parameters in addition to the two classical Lame constants [12–14] in

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the constitutive equations. As some applications, Zhou and Li [15] employed this theory to investigate the static and dynamic torsion of a micro-bar. Also, the size-effects in Timoshenko beams modeled on the basis of the couple stress theory have been investigated by Asghari et al. [16].

Yang et al. [17] argued that in addition to the classical equilibrium equations of forces and moments of forces, another equilibrium equation should be considered for the material elements. This additional equation is the equilibrium of moments of couples. Then, they concluded that this additional equilibrium equation implies the symmetry of the couple stress tensor. Accordingly, they modified the constitutive equations of the couple stress theory and present the new constitutive equations with only one material length scale parameter.

In order to determine the length scale parameter l for a specific material, some typical experiments such as micro-bend test, micro-torsion test and specially micro/nano-indentation test can be carried out (see [7,8,10,18–20]).

The modified couple stress theory has been utilized to develop the size-dependent formulations for beams by some researchers. For example, Park and Gao [21] analyzed the static mechanical properties of an Euler–Bernoulli beam modeled on the basis of the modified couple stress theory and interpreted the outcomes of epoxy polymeric beam bending test. Also, Kong et al. [11] studied the natural frequencies of the beam based on the modified couple stress theory. In addition, the size-dependent natural frequencies of fluid-conveying microtubes [22], the size-dependent buckling behavior of micro-tubules [23] and the size-dependent resonant frequencies and sensitivities of AFM microcantilevers [24] have been investigated based on the modified couple stress theory. A new Timoshenko beam model based on the modified couple stress theory was formulated by Ma et al. [25]. They assessed the size-dependent static and free-vibration behavior of a simply-supported Timoshenko beam as a case study.

In beams used in MEMS and NEMS with two immovable supports, we face with the nonlinear phenomena in the large amplitude deflections. The source of the nonlinearity is the induced mid-plane stretching during the transverse deflections. This nonlinearity causes the static and vibration results to be changed significantly [2,26–28]. Hence, the abovementioned linear investigations on the couple stress beams are not appropriate in these conditions. Recently, a size-dependent nonlinear Euler–Bernoulli beam model has been presented by Xia et al. [29] on the basis of the modified couple stress theory. They studied the nonlinear size-dependent static bending, buckling and the free vibration of beams.

Considering the presentation of the linear modified couple stress Euler–Bernoulli beam [11], the linear modified couple stress Timoshenko beam [25], and the nonlinear modified couple stress Euler–Bernoulli beam [29] formulations, our goal in this paper is the establishing of the nonlinear modified couple stress Timoshenko beam formulation as the next step in the sequential mentioned works. This paper rigorously derive the governing equations and boundary conditions for the modified couple stress based nonlinear Timoshenko beam. The formulation of the manuscript is theoretically more complicated and consequently possesses a more theoretical value. On the other hand, this complexity makes it capable to produce more appropriate results and simulations, as will be discussed in the following.

The derived formulation of the manuscript enjoys the following three properties together, for the first time in the literature:

(A) The size of elements in micro- and nano-electromechanical systems (MEMS and NEMS) is very small and as a result, using non-classical continuum theories such as the modified couple stress theory for modeling the material behavior is crucial for getting appropriate and accurate results in analyzing or designing these elements. Accordingly, in recent years many works have been presented in the literature based on the modified couple stress theory (e.g. [15,16,21–24]). In other words, the experimentally validated small scale effects [11–14] are very important in MEMS and NEMS elements and classical theories are not capable to treat them appropriately.

(B) It has experimentally been observed that the effects of nonlinearities are very significant on the behavior of micro- and nanomechanical resonators, even in not so much large amplitudes [30–32]. Hence, using the nonlinear formulations for simulating and designing of such MEMS devices seems to be essential such that many researchers have been attracted to study the nonlinear effects. As an example, Zhang et al. [33] has shown, both analytically and experimentally, that the nonlinearities have a large impact on the dynamic response of the micro-resonant oscillators. As another example, Adams et al. [34] have investigated nonlinear effects on the harmonic resonance of these elements.

(C) Utilizing the Timoshenko beam, the effects of shear deformation and the rotary inertia are included in the formulation, while these effects are ignored in Euler–Bernoulli beams. These effects are not negligible in even thin beams that are vibrating at high frequencies. Due to their dimensions, resonance frequencies of micro- and nano-scale resonators are extremely high, and consequently modeling them by Timoshenko model have great merits over the Euler–Bernoulli model to produce accurate and reliable results. As an example, the existence of the resonance frequency due to the shear deformation have been experimentally validated by Barr (see [35]), while the Euler–Bernoulli model is not capable to predict it, but the Timoshenko theory is.

Because of possessing the three mentioned important properties, the established formulation of this paper is more appropriate than the available formulations. Using the derived formulations, the nonlinear size-dependent static and free-vibration behaviors of a hinged–hinged micro-beam are assessed. The obtained results are compared with those corresponding to the linear modified couple stress theory as well as linear and nonlinear classical theories.

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