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Synchronization of complex networks with asymmetric coupling via decomposing matrix method

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ABSTRACT

In this paper, the problem concerning synchronization is investigated for complex networks with time delay and asymmetric coupling. By decomposing the asymmetric coupling matrix and employing Lyapunov functional method, sufficient conditions are obtained for synchronization. Finally, two examples are reported to illustrate the effectiveness of some proposed methods.

1. Introduction

Complex networks lie in many fields of our daily life, such as the Internet, World Wide Web, communication networks, social networks, genetic regulatory networks, power grid networks, and so on (see [5,19,20,25]). The word synchronization comes from Greek, which means share time .Synchronization of complex networks of dynamical systems has received a great deal of attention from the nonlinear dynamics community. In the past decades, special attention has been focused on the synchronization of chaotic dynamical systems, particularly large-scale and complex networks of chaotic oscillators [2,7,26,27]. Recently, synchronization in different small-world and scale-free dynamical network models has also been carefully studied [1,22,28,29]. In recent years, due to wide applications, the problem on synchronization in various dynamical networks has been extensively studied in present literature see e.g. [6,8–11,13–16,21,24,30,31,33,35] and references therein. These studies may shed some new lights on the synchronization phenomenon in various real-world complex networks.

In real world situation, time delay is ubiquitous in many physical systems due to the finite switching speed of amplifiers, finite signal propagation time in networks, finite reaction times, memory effects and so on. Furthermore, the time delay may cause undesirable dynamic behaviors such as oscillation, instability and poor performance. Therefore, the synchronization problem of complex dynamical networks with time delays has become a topic of both theoretical and practical importance. In the past decades, many researchers have contributed to the area of synchronization of delayed complex networks. In [23], Song has studied cluster synchronization problem for an array of coupled stochastic delay neural networks via pinning control strategy. Synchronization of edge-colored networks was studied from adaptive control and pinning control approaches [32]. Deng et al. in [3] investigated synchronization of a complex network with the non-derivative and derivative coupling. Both linear and adaptive feedback control methods are utilized to design controller. The problem of function projective synchronization have been obtained for general complex dynamical networks with time delay via hybrid feedback control method [4].

Some previous studies have been conducted on the following two cases: (i) linearly (or nonlinearly) coupled complex networks (i.e., the coupling function is linear (or nonlinear)) [4,17,18]; (ii) undirected interaction topology (i.e., the coupling matrix should be symmetric or irreducible)[36]. However, very little is known about the complex networks with asymmetric coupling matrix[12,34], which are important in practical applications, for instance, via broadcasting. In realistic, the influences should be different to each other, and hence the coupling configuration matrices of networks should be restructured in a more general asymmetric form. Compared with [12,34], we will utilize the matrix splitting method to deal with asymmetric coupling.

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Motivated by the above discussions, in this paper, we will study the synchronization of complex dynamical networks with time delay and asymmetric coupling. We decompose asymmetric matrix into two matrices, use some sufficient methods to deal with coupling matrix and employ Lyapunov functional. So some sufficient conditions for synchronization of complex networks with time delay and asymmetric coupling are derived. Therefore, the conservatism of synchronization criteria is reduced. Finally, examples are given to illustrate the effectiveness of the proposed methods. It is remarked that this paper differs from the works in [3,17,18,30,32] in mainly the following points: (1) they did not consider time delay linear item in [3,17,18,30,32]; (2) the coupling matrix is assumed to be symmetric in [3,17,18,30,32], here, we reduce constraint about the coupling matrix, the coupling matrix is defined as asymmetric. When $\tau = 0$, the results of [17,18,30,32] are the special case of Corollary 3.1; (3) they required the coupling matrix that $\| A \| = \gamma > 0$ and $\|A_\tau\| \le k$ in [30], we need not these conditions.

The remainder of the paper is organized as follows. The network model is introduced and some necessary lemmas are given in Section 2. Section 3 discusses synchronization of the complex dynamical networks with time delay and asymmetric coupling matrix. The theoretical results are verified numerically by several representative examples in Section 4. Finally, this paper is concluded in Section 5.

Throughout this paper, $\Re^n$ denotes n-dimensional Euclidean space and $\Re^{n\times n}$ is the set of all $n \times n$ real matrices. For symmetric matrices X and Y, the notation $X > Y$ ($X \ge Y$) means that the matrix $X - Y$ is positive definite (nonnegative).

2. Preliminaries

Consider a complex dynamical network consisting of N identical nodes with asymmetric coupling:

$$\dot{x}_i(t) = f(x_i(t)) + Ax_i(t-\tau) + c\sum_{j=1}^{N} g_{ij}\Gamma x_j(t), \tag{1}$$

where $i = 1, 2, \dots, N$, $x_i(t) = (x_{i1}(t), \dots, x_{in}(t))^T \in \Re^n$ is the state vector of node i, f: $\Re^n \to \Re^n$ is continuously differentiable. $c > 0$ is the coupling strength, $\Gamma = diag(\gamma_1, \dots, \gamma_n)$ is a nonnegative matrix. $G = (g_{ij})_{N\times N}$ is the coupling matrix satisfying $\sum_{j=1}^{N} g_{ij} = 0$. The off-diagonal elements of G are not assumed to be nonnegative.

Definition 2.1. The dynamical networks (1) are said to achieve synchronization if

$$\lim_{t\to\infty} \|e_i(t)\| = \lim_{t\to\infty} \|x_i(t) - s(t)\| = 0, \quad i = 1, 2, \dots, N,$$

where $\|\cdot\|$ stands for the Euclidean vector norm and $s(t) \in \Re^n$ is a solution of an isolate node, satisfying $\dot{s}(t) = f(s(t)) + As(t-\tau)$.

Define the error vectors as

$$e_i(t) = x_i(t) - s(t) \qquad (i = 1, 2, \dots, N). \tag{2}$$

Then, the following error dynamical network can be obtained:

$$\dot{e}_i(t) = f(x_i(t)) - f(s(t)) + Ae_i(t-\tau) + c\sum_{j=1}^{N} g_{ij}\Gamma e_j(t), \tag{3}$$

Assumption 1 (QUAD). There exists a positive definite diagonal matrix $Q = diag(q_1, \dots, q_n)$, a diagonal matrix $\Delta = diag(\delta_1, \dots, \delta_n)$ satisfying $\delta_j \ge 0$ for $j = 1, \dots, n$, and a constant $\varepsilon > 0$, such that

$$(u - v)^T Q[f(u) - f(v) - \Delta(u - v)] \le -\varepsilon(u - v)^T(u - v)$$

holds for any u, $v \in \Re^n$.

Assumption 2. $g_{ij} + g_{ji} \ge 0$, $i, j = 1, \dots, N, i \ne j$.

Lemma 2.1. *Let* $\forall x$, $y \in \Re^n$, *then*

$2x^T y \le x^T x + y^T y.$

3. Main results

In this section, the synchronization of complex network with delay and asymmetric coupling is considered.

Theorem 3.1. *Assume that the Assumptions 1 and 2 hold. For the complex system (1)can achieve synchronization if there exists diagonal matrices* $P > 0$, $Q > 0$, *and* ε *is sufficiently large positive constant, such that the following inequalities hold:*

$$\varepsilon \ge \frac{1}{2} + q_k\delta_k + cq_k\gamma_k\lambda_{max}(\breve{G}) + p_k \tag{4}$$

and

$$\lambda_{max}(QAA^TQ^T) \le \lambda_{max}(P) \tag{5}$$

where $\lambda_{max}(\breve{G})$ *is the largest eigenvalue of* $\breve{G}$ *and* $k = 1, \dots, N$.

Proof. Construct a Lyapunov functional as follows:

$$V(e_t) = \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)Qe_i(t) + \sum_{i=1}^{N}\int_{t-\tau}^{t} e_i^T(s)Pe_i(s)ds. \tag{6}$$

The derivative of $V(e_t)$ along the trajectories of Eq. (3) is

$$\begin{aligned}
\dot{V}(e_t) &= \sum_{i=1}^{N} e_i(t)Q\dot{e}_i(t) + \sum_{i=1}^{N} e_i^T(t)Pe_i(t) - \sum_{i=1}^{N} e_i^T(t-\tau)Pe_i(t-\tau) \\
&= \sum_{i=1}^{N} e_i^T(t)Q\left[f(x_i(t)) - f(s(t)) + Ae_i(t-\tau) + c\sum_{j=1}^{N} g_{ij}\Gamma e_j(t)\right] \\
&\quad + \sum_{i=1}^{N} e_i^T(t)Pe_i(t) - \sum_{i=1}^{N} e_i^T(t-\tau)Pe_i(t-\tau) \\
&= \sum_{i=1}^{N} e_i^T(t)Q\Bigg[(f(x_i(t)) - f(s(t)) - \Delta e_i(t)) \\
&\quad + (\Delta e_i(t) + Ae_i(t-\tau) + c\sum_{j=1}^{N} g_{ij}\Gamma e_j(t))\Bigg] \\
&\quad + \sum_{i=1}^{N} e_i^T(t)Pe_i(t) - \sum_{i=1}^{N} e_i^T(t-\tau)Pe_i(t-\tau).
\end{aligned} \tag{7}$$

According to Assumption 1, we get

$$\begin{aligned}
\dot{V}(e_t) &\le -\varepsilon\sum_{i=1}^{N} e_i^T(t)e_i(t) \\
&\quad + \sum_{i=1}^{N} e_i^T(t)Q\left[\Delta e_i(t) + Ae_i(t-\tau) + c\sum_{j=1}^{N} g_{ij}\Gamma e_j(t)\right] \\
&\quad + \sum_{i=1}^{N} e_i^T(t)Pe_i(t) - \sum_{i=1}^{N} e_i^T(t-\tau)Pe_i(t-\tau)
\end{aligned} \tag{8}$$

Using Lemma 1, we can obtain

$$\begin{aligned}
\dot{V}(e_t) &\le -\varepsilon\sum_{i=1}^{N} e_i^T(t)e_i(t) + \frac{1}{2}\sum_{i=1}^{N} e_i^T(t)e_i(t) \\
&\quad + \frac{1}{2}\sum_{i=1}^{N} e_i^T(t-\tau)QAA^TQ^Te_i(t-\tau) \\
&\quad + \sum_{i=1}^{N} e_i^T(t)Pe_i(t) - \sum_{i=1}^{N} e_i^T(t-\tau)Pe_i(t-\tau) + M.
\end{aligned} \tag{9}$$

where $M = \sum_{i=1}^{N} e_i^T(t)Q[\Delta e_i(t) + c\sum_{j=1}^{N} g_{ij}\Gamma x_j(t)]$.