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Determining the embedding parameters governing long-term dynamics of copper prices

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ABSTRACT

Mineral extraction activities and the commercialisation of mineral-based products in later stages are fundamental for economic, technological and social development, so understanding their long-term market behaviour in order to forecast prices is crucial for governments, companies and society. Mineral commodity markets are multidimensional as they are driven by the dynamic evolution of financial, technological, psychological and geopolitical factors. Thus, identifying the key features governing mineral commodity prices and modelling their long-term behaviour is an intricate problem that becomes even more complex when the relationship with time is also considered. Although several techniques are available to represent the behaviour and reduce the dimensionality of data sets, these techniques often neglect the temporal relationship and evolution of variables. In many cases, the selection of key variables is solely based on the correlation with each other, relevance and degree of variance, and the cause and effect behaviour of the time-relation of variables, is neglected. This is a critical issue in assessing dynamic systems, especially those involving human behaviour such as social sciences and biology where learning and cognition capacities evolve through time. In this sense, before any attempt to forecast mineral commodity prices, a proper understanding of the embedding dimension and time delay of the variables governing the system is fundamental to determining the number of key features driving the system and the extension of the delayed effects of changes in the initial conditions, respectively. Determining these parameters may become even more complicated because of data scarcity, since the high cost, technical or temporal limitations often hamper obtaining large data sets related to complex dynamic systems that occur in nature and the social sciences. Daily and monthly quotations have been commonly used to represent the dynamics of mineral commodity markets and predict prices, yet they are not sufficient to capture and understand the dynamic evolution of prices in the long-term. These quotations are too short as the delayed effects that reflect on the entire economy and arises from changes in macroeconomic variables can normally be perceived about a year ahead, and may last for up to six years. However, chaos theory has provided valuable tools to assess the dynamics of complex systems allowing the proper determination of the embedding parameters (dimension and time delay), even when using small data sets. This paper examines the long-term dynamics of mineral commodity prices via chaos theory based on a short data set of annual copper prices observed between 1900 and 2015. Copper was used as a study case because it is one of the most competitive markets and representative of mineral commodities traded worldwide. It was found that the dynamic-deterministic behaviour of annual copper prices is embedded in a high-dimensional space that fluctuates between five and seven, and a relatively short time delay between two

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and three periods. Our finding argues for the use of chaos theory as an important technique to assess the long-term behaviour of mineral commodity prices using short data sets because it improves the understanding of those complex dynamic systems and provides important guidelines that remarkably simplifies the forecasting task.

1. Introduction

Mineral commodity markets are significantly influenced by human behaviour and, therefore, are considered complex dynamic systems due to the large number of economic, financial, political and psychological variables that may fluctuate over time [1]. However, the evolutive and time-related nature of these factors in the economy has not been extensively assessed for their role in forecasting market prices [2–6], since econometric, stochastic and time series techniques have been the traditional [7] means for forecasting. Econometric models can correlate many variables [8,9] but they are static and based on the belief that past events will repeat in the future [10]. Stochastic models can solve problems of continuous dynamic systems and include uncertainties by using discrete-time approximations where variables fluctuate within pre-established boundaries [11,12]. However, these models are unable to manage the vast amount of information required to accurately forecast prices and represent effectively the relationship between variables [10,13]. Moreover, although economic theory asserts the balance of markets only in the long-term, prices and costs do not fluctuate randomly [14]. Time series can represent price fluctuations and volatility, but they are detected by merely measuring the "unusual deviation of errors"; therefore, their causes and effects are unknown, and cannot be tracked over time [10].

Even if these models could properly mimic the dynamic behaviour of mineral commodity prices by correlating variables through statistical techniques, there are four debatable questions that they cannot answer. (1) What is the number of variables that should be assessed and which are they? (2) How far into the past must researchers look to understand the evolution of the variables governing the system and for how long do the effects of these variable remain in the systems? (3) What is the appropriate time span to determine if unusual deviations are indeed anomalies or simply an evolutive pattern? (4) Do the changes in variables have stochastic origins and, although human beings drive the markets, does the system itself evolve randomly? From these largely unanswered questions, a lack of knowledge concerning temporal relationships between the variables considered in these techniques to describe the cause and effect evolution of patterns is evident. Nonlinear behaviour in prices, and sensitivity to the initial condition have been found in several markets [3,9,13,15–18]. This, along with the unanswered questions listed above, undermines the adequacy of traditional econometric, stochastic and time series techniques to represent the complex dynamic behaviour of markets. Hence, to increase the accuracy of mineral commodity price forecasting, the temporal relations of variables embedded in a specific and well-established dimension should be considered by these techniques to either improve their performance or discard them.

Sensitivity to initial conditions, temporal relationships, self-similarity patterns and deterministic features of nonlinear dynamic systems were formalised by the Chaos Theory in 1963 [1,19–26]. Chaos theory allows the assessment of the interaction among variables embedded in high dimensional spaces and discover complex patterns governing the behaviour of systems without the need to evaluate each potentially relevant variable. As a result, it can accurately detect patterns of deterministic dynamic processes that may appear stochastic and cope with data scarcity since it is able to discriminate between deterministic patterns and genuine random systems that, due to the absence of long data sets, cannot be properly described and assessed by other techniques [18,23,26–29]. Patterns are detected by the presence of a strange attractor confined (attracted) to one specific, limited area of a reconstructed phase space within an embedding dimension (m) and time delay (τ) [15,22,25,26,30–32], which are fundamental to predicting future states of the system. As chaotic behaviour has been detected in many complex systems involving biological, physical, economic and market phenomenon [13,17,31,33–45], even when the data sets are short, chaos theory appears as a novel and reliable approach for assessing the behaviour of mineral commodity prices.

The chaotic dynamics of tin, zinc, crude oil and its subproducts as well as the nonlinear long-term memory dynamics of aluminium and copper prices [11,28,46–51] have been demonstrated in earlier studies, yet their nonlinear dynamics features have been assessed using daily or monthly quotations limited to sample periods of up to 15 years only. Nevertheless, it is important to note that changes in macroeconomic variables such as interest rates, monetary policy, taxes, federal debts and investment may have effects on the economy at least three quarters ahead and these effects may remain for up to six years during financial crises [2,52–62]. Therefore, the time-related behaviour of mineral commodity prices in the long-term should be examined using long-term data sets on an annual basis [2,52–61,63]. It is crucial to assure that the temporal relation of the system is contained within a proper time delay (τ) which will be used to determine an adequate embedding dimension (m) and obtain a realistic and reliable representation of the long-term dynamics of mineral commodity price systems. In other words, the temporal space between observations should be long enough to capture the temporal evolution of the long-term effects that the changes in the variables induce in the economy and markets.

While is true that the more number of years investigated, the better the understanding of the evolution of complex dynamic systems, acquiring long data sets is not always feasible due to cost, technical or temporal limitations [64–66]. The latter is an important restriction for time series of market price evolution, including that of mineral commodities, as the length of long-term (annual base) data sets are limited to the time and date when market transactions were established [64]. Data scarcity has been a much-debated issue in the study of long-term behaviour of mineral commodity prices and yet, their features have been assessed using small data sets of annual quotations containing between 107 and 137 observations [11,67–70]. Although none of these studies investigate the dynamic behaviour of prices, the temporal evolution of variables, the main features driving the systems or the time delay of effects, they were useful to settle the time frame for this type of studies. In a different approach. Rossen [71] used 1024 monthly observations to assess the long term dynamics of several mineral commodity prices. Though the use of large data sets is remarkable, monthly quotations are small compared to the spectrum covered by the temporal influence that changes in macro economic variables have on both the global economy and mineral commodity markets and prices in the long term.

In this manuscript, copper was investigated as a study case and it was verified that average yearly basis prices traded in open markets are available only from 1877 which was the year of founda-