Chaos, Solitons and Fractals 111 (2018) 157–168

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Frontiers

Control, electronic circuit application and fractional-order analysis of hidden chaotic attractors in the self-exciting homopolar disc dynamo

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ARTICLE INFO

Article history:
Received 24 November 2017
Revised 4 February 2018
Accepted 12 April 2018

Keywords:
Homopolar disc dynamo
Hidden attractor
Chaos control
Circuit application
Fractional order chaotic system
FPGA realization

ABSTRACT

Based on the segmented disc dynamo proposed by H. K. Moffatt, we give out the hidden chaotic attractors, which can show the imperfection in the dynamo model. In this paper. control of hidden chaos in the model is investigated by Lyapunov based nonlinear feedback controllers, sliding mode controllers and hybrid combination of them. Numerical simulations on the comparative analyses are presented. Moreover, with the aid of ORCAD-Pspice and oscilloscope, hidden chaos can be implemented by electronic circuit. Compared with the phase portraits using MATLAB, the simulation results of the oscilloscope outputs verify the effectiveness of electronic circuit design. Finally, in order to consider the effects of fractional order, we analyze the fractional order chaotic system (FOCS) and consider its FPGA implementation for the self-exciting homopolar disc dynamo. The results are helpful for us to better understand the dynamics behavior of disc dynamos.

1. Introduction

The research of chaotic systems is an important and yet difficult task in the disciplines of nonlinear dynamics. The dynamical mechanism of chaotic attractors with unstable equilibria can be applied to designing complex hyperchaotic attractors and communication encryption [51]. From another perspective, Hidden periodic states arose in the analysis of Hilbert's 16th problem on the number and location of limit cycles in systems with quadratic nonlinearities [23]. Hidden oscillations can be obtained as a natural feature in electromechanical rotational models and electrical circuits [7,18,52].

Recently, In order to consider dynamical systems with polynomial nonlinearities, oscillations and general issues of stability, etc., the idea of a "hidden attractor" was found in the mid 20th century [33]. Hidden chaotic attractors or hyperchaotic attractors have stimulated a lot of research interest ([25,28,30,31,61,63,64,67]; Wei et al., 2015; [65,66]; Wei et al., 2015). In comparison, hidden chaotic attractors from nonlinear systems with a stable equilibrium, or a nonlinear system without equilibria, are not easy to be localized because the attraction basin for the hidden attractor does not intersect with any small neighborhoods of any equilibria [33–36].

It was traditionally believed that chaotic motions were uncontrollable due to their extreme sensitivity. Researchers frequently encounter chaos control in chaotic systems with engineering background due to the sensitivity of chaos to initial values. The purpose of control is to suppress chaotic motions, and drive them towards one of the system's stable steady states. Hubler [24] showed that control of chaotic systems can be achieved. Then, Ott et al. [40] introduced a chaos control method named OGY. Since these pioneering studies, many researchers have paid attention to control of chaos, such as linear feedback control [70], nonlinear control [2,5,11,19,39,41,43], adaptive control [58], sliding mode control ([71]; Jang et al., 2000; [1,10,38,72]; Kocamaz et al., 2009; [57]), active nonlinear control (Li et al.,2009; [59]), passive control [12] and backstepping design [44].

As mentioned in the work by Plunian [45], dynamos will be usefull to understand magnetic field generation and reversals in astrophysical bodies. The Bullard model [8], known mainly for its educational interest, presents the typical features of a fluid dynamo. The conventional description of the one-disc dynamo has been shown to be misleading from the fundamental point of view. H. K. Moffatt in 1979 [37] proposed a self-exciting homopolar

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https://doi.org/10.1016/j.chaos.2018.04.020
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dynamo, which segmented the disc by insulating foils, in such a way that the current is forced to flow radially, except in the neighbourhood of the disc's rim, where it is azimuthal. The dynamo has not yet been completely well understood because of the existence of hidden chaotic attractors, which is one of important features for imperfect systems. Because hidden chaos is belong to one of the imperfect uncertain systems [20,21], the goal of control engineering is to take suitable feedback actions despite imperfections and uncertainties in the model for disc dynamo.

Our objective is to investigate control, electronic circuit application and fractional-order analysis self-exciting homopolar disc dynamo in the paper. Well-known nonlinear based controllers and sliding mode controllers are assigned for controlling the hidden chaos. In order to view the hidden chaos in real time, we design an electronic circuit, and use an oscilloscope in the re-scaled dynamo. Finally, fractional order self-exciting homopolar disc dynamo and its FPGA implementation for the self-exciting homopolar disc dynamo are analyzed.

2. Hidden attractors in self-exciting homopolar disc dynamo

2.1. System description

Dynamo models have been established to discuss the generation of magnetic fields and their reversals in astrophysics. Moffatt [37] extended the simplest self-exciting Bullard dynamo to include radial diffusion of magnetic field, and obtained the segmented disc dynamo model, which was written non-dimensionally as:

$$\begin{cases} \dot{x} = r(y - x) \\ \dot{y} = mx - (1+m)y + xz \\ \dot{z} = g[1 + mx^2 - (1+m)xy]. \end{cases} \tag{1}$$

Here $x(t)$ and $y(t)$ denote the magnetic radial and azimuthal current distributions, $z(t)$ is the angular velocity of the disc, and the dot denotes differentiation with respect to time; g measures the applied torque, and r and m are positive constants that depend on the electrical properties of the circuit.

From the following equations

$$\begin{cases} r(y - x) = 0, \\ mx - (1+m)y + xz = 0, \\ g\big[1 + mx^2 - (1+m)xy\big] = 0, \end{cases} \tag{2}$$

we can obtain two equilibria: $E_1(-1,\ -1,\ 1)$ and $E_2(1,\ 1,\ 1)$. Linearizing the system (1) about $E_{1,\,2}$yields the following characteristic equation:

$$2gr + (g + gm)\lambda + (1 + m + r)\lambda^2 + \lambda^3 = 0. \tag{3}$$

Base on the Routh–Hurwitz criterion, $E_{1,\,2}$are both locally asymptotically stable if and only if

$$m \geq 1 \text{ or } m < 1, r < \frac{(m+1)^2}{1-m}. \tag{4}$$

In order to consider hidden chaos in the self-exciting homopolar disc dynamo (1), we firstly give out the comment from Prof. Moffatt: "When $r < \frac{(m+1)^2}{1-m}$, it seems probable that all trajectories tend to one of the two equilibrium points" [37]. However, as we know, multistability depends on the choice of initial conditions, as well as small changes in parameters, so that a sudden transition can occur to a different attractor. Hidden chaos, called one kind of imperfection, can actually exploited to reveal complex dynamics.

2.2. Hidden chaos

We now address multistability for Moffatt' inequalities:

$$\{(m, r) | m < 1, r < \frac{(m+1)^2}{1-m}\}. \tag{5}$$

When $r = 10$, $m = 0.75$, $g = 20$, we can obtain chaotic attractor, which comes from the effects of different choice of initial data.

(A) For initial values of (2, 2, 0.75), we obtain a hidden chaotic attractor with Lyapunov exponents: $L_1 = 0.5145$, $L_2 = 0.0000$, $L_3 = -12.2685$. It shows the coexistence of chaos with stable steady states;
(B) When we choose initial conditions (1.5, 1.5, 0.75), chaotic attractors can not be obtained and Lyapunov exponents: $L_1 = -0.0332$, $L_2 = -0.0332$, $L_3 = -11.6836$.

The above Lyapunov exponents are calculated using Wolf's method to run the orbit for a time of 4e7 using a fourth-order Runge-Kutta integrator with an adaptive step size. They can be obtained from a standard Gram-Schmidt reorthonormalization procedure, using a Jacobian matrix [22,32,68]. Note that the behavior of system (1) is very sensitive to initial values. A change in the initial values for x and y lead to very different dynamics. Fig. 1 shows these two coexisting attractors, which show orbit of system (1) will eventually evolve into either a stable steady state or a chaotic attractor. This suggests there are still discoveries to be made towards a unified theory for chaotic systems [62]. In this following sections, we mainly focus on the control, electronic circuit application and fractional-order analysis for the self-exciting homopolar disc dynamo (1) .

3. Control for the self-exciting homopolar disc dynamo (1)

In order to drive the chaos of system (1) towards its equilibria control signals u_1, u_2 and u_3 are added to the x, y and z state variables of the system, respectively:

$$\begin{cases} \dot{x} = r(y - x) + u_1, \\ \dot{y} = mx - (1+m)y + xz + u_2, \\ \dot{z} = g\big[1 + mx^2 - (1+m)xy\big] + u_3. \end{cases} \tag{6}$$

We represent an equilibrium as (x_d, y_d, z_d), and define perturbations about this state as error states: $e_1 = x - x_d, e_2 = y - y_d$and $e_3 = z - z_d$. Thus, the equations for the errors (e_1, e_2, e_3) become:

$$\begin{cases} \dot{e}_1 = r(e_2 - e_1 + y_d - x_d) + u_1, \\ \dot{e}_2 = (m + z_d)e_1 - (1+m)e_2 + x_d e_3 + e_1 e_3 + m x_d \\ \qquad - (1+m)y_d + x_d z_d + u_2, \\ \dot{e}_3 = g\big[1 + m(e_1^2 + 2x_d e_1 + x_d^2) \\ \qquad - (1+m)(y_d e_1 + x_d e_2 + e_1 e_2 + x_d y_d)\big] + u_3. \end{cases} \tag{7}$$

Since $y_d - x_d = 0$, $mx_d - (1+m)y_d + x_d z_d = 0$ and $1 + mx_d^2 - (1+m)x_d y_d = 0$, the system (7) can be simplified as

$$\begin{cases} \dot{e}_1 = r(e_2 - e_1) + u_1, \\ \dot{e}_2 = (m + z_d)e_1 - (1+m)e_2 + x_d e_3 + e_1 e_3 + u_2, \\ \dot{e}_3 = g\big[m(e_1^2 + 2x_d e_1) - (1+m)(y_d e_1 + x_d e_2 + e_1 e_2)\big] + u_3. \end{cases} \tag{8}$$

Now we use different control methods to produce asymptotically stable states of system (8) at the zero point.

3.1. Nonlinear feedback control

We introduce the Lyapunov function

$$V(e_1, e_2, e_3) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2) \tag{9}$$