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Fractional-order modeling and dynamic analyses of a bending-torsional coupling generator rotor shaft system with multiple faults

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ABSTRACT

Unexpected vibrations induced by the crack fault and other unbalance factors in rotor system seriously affect the health and reliability of the generator. Here, to explore the vibration performances, a bending-torsional coupling model of the generator rotor shaft system is established, in which electromagnetic malfunction (unbalanced magnetic pull) and mechanical failures (fractional-order damping, crack and contact-rubbing) are considered. Then, the simulation is done by a modified Adams-Bashforth-Moulton algorithm. Based on the simulation, the correctness of the new coupling model is verified by comparing with previous model and experimental data. At the same time, the new coupling model is analyzed to obtain the dynamic evolutions of the generator rotor shaft system with the changes of crack depth ratio, the fractional order of damping, rotational speed ratio and mass eccentricity of rotor. In addition to this, some critical values and ranges are proposed. Finally, these results can efficiently provide a theoretical reference for the design of generator rotor system and be applied to forecasting and diagnosing vibration faults in generator rotor shaft system.

1. Introduction

With the fast development of manufacture and energy project, a multitude of rotational machinery with larger torque, higher rotational speed, and lighter shafts have been designed and applied to practice engineering [1–3]. However, these changes increase the susceptibility of all kinds of faults. Further, these faults will introduce the intense vibration and even disastrous accident. Recently, the massive generator, as a rotational machinery, has been paid close attention to, and it is found that there are kinds of troubles in operation including shaft fatigue cracking, rotor-stator contact, the mass eccentricity of rotor, and asymmetric electromagnetic excitement [4–6]. These troubles could lead to bending and torsional vibrations of the generator rotor shaft system (GRSS), which not only reduce generating efficiency of the system, but could cause wear of rotor and stator and even shaft rupture accident. A representative case is the accident happening in Sayano–Shushenskay hydropower station in 2009 [7].

Many researchers have intensively studied the dynamic characteristics of the all kinds of rotational machineries from different viewpoints. Some of them mainly paid close attention to crack fault. Gash [8] presented a comprehensive survey of simple transverse crack rotors, which is restricted to a Laval rotor. Darpe et al. [9] studied the unbalanced response and coupling characters of lateral and longitudinal vibrations of a cracked Jeffcott rotor with periodic axial impulses. Sinou et al. [10] analyzed the influence of transverse crack in a rotating shaft. A contrastive analysis model was established by Patel et al. [11] to explore the dynamic differences between switching crack model and breathing crack model. Ebrahimi et al. [12] analyzed the forced vibration of rotors with an open edge crack from time and frequency domains. Using empirical mode decomposition (EMD) method, Yang et al. [13] established a model to forecast the cracked rotor's nonlinear response. Babu et al. [14] applied Hilbert-Huang transform (HHT) to realize detection and monitoring of crack in a transient rotor. Kulesza [15] evaluated the influence of the crack depth in a rotor-bearing system using multisine technique. Upadhyay et al. [16] established a new improved theoretical model of the rotor-bearing system and analyzed the dynamic behavior of the system due to the transverse crack. At the same time, some scholars mainly concerned contact-rubbing fault between rotor and stator. Muszyska [17] provided a review of the literature on contact-rubbing fault. Chu et al. investigated nonlinear vibration characteristics of a typical rub-impact Jeffcott rotor [18], and then who also studied the nonlinear vibrations of a rub-impact rotor system by experimental observation

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Table 1
Differences between the new model and the previous models.

	New model	Previous models
1	Considering the complex actual engineering conditions, including the electromagnetic malfunction (unbalanced magnetic pull) and mechanical failures (crack and contact-rubbing).	Researching the dynamic characteristic of the rotor shaft system in an ideal condition, and mainly considering the single crack fault (Ref. [11]), contract-rubbing fault (Ref. [21]) or electromagnetic malfunction (Ref. [36]).
2	A coupled bending-torsional vibration model is presented.	A single bending vibration model was applied to (Ref. [37]).
3	Considering the history memory characteristic of the damping, the fractional-order calculus is applied to describing the damping characteristic more precisely.	The damping characteristic was described by the simple integer-order calculus (Ref. [37]).

[19]. Qin et al. [20] investigated the chaotic response and bifurcation of a rub-impact overhung rotor in the oxygen pump by phase diagram. The chaotic vibration analysis of a rotating flexible continuous shaft-disk system with rub-impact was discussed by Khanlo et al. [21]. Using modern nonlinear dynamics and bifurcation theories, Zhang et al. [22] established and analyzed a Jeffcott microrotor rub-impact system based on the classic impact theory. The nonlinear dynamic behavior of the rub-impact rotor-bearing system supported by oil-film short bearings was studied by Chang-Jian et al. [23]. Mokhtar et al. [24] investigated the rotor-stator rub-impact phenomenon in the finite element framework using Lagrange multiplier based contact mechanics approach. In addition, there are a little works focusing on the dynamic characteristics of the nonlinear coupled, multi-fault forces interaction, and other more complex models. Ren et al. [25] studied the dynamic characteristics of multiple-degree-of-freedom system rotor-bearing system with coupling faults. Pennacchi et al. [26] explored the character of short arc rub by a real multi-rotor machines experiment. Using Lagrangian equation, Yuan et al. [27] established a complete six degree of freedom rotor model to demonstrate the dynamic behaviors of the rotor by means of some nonlinear dynamic methods. Khanlo et al. [28] investigated the lateral-torsional coupling effects on the nonlinear dynamic behavior of a rotating flexible shaft-disk system.

In light of the above analysis, first, these works, both simulation and experiment, mainly focus on the dynamic characteristic of a rotor shaft system under an ideal condition, which may be incommensurate to the actual engineering condition for the existence of complicated coupling relationship. Therefore, it is necessary for a specific engineering to establish an appropriate model according to its idiographic nature and operation condition. Besides, previous analyses mainly paid attention to the bending vibration of rotor shaft system and ignored the torsional vibration, while the torsional vibration is a universal character of the massive rotational machinery such as hydro-generator. Hence, a coupled model needs to be established to analyze the bending-torsional coupling vibration characteristic. Finally, most of these models mentioned in previous are based on integer-order calculus, which emphasizes local character. On the contrary, the fractional-order operator has a nonlocal characteristic, which has been successfully applied in many fields [29–33]. Concretely, fractional-order damping is a typical application in the analyses of rotating machinery [34,35]. In light of the earlier discussions, Table 1 is presented to clarify the differences between previous models and the new model from modeling methods.

Motivated by the above analysis, comparing with the previous works, there are three advantages for our research works. First, the fractional derivative is introduced into the modeling of rotating machinery. Second, based on engineering practice, an electromechanical coupling model of the generator crack rotor shaft system (GCRSS) with coupled bending-torsional vibration is established. Third, the stability of GCRSS with the changes of crack depth ratio (A), the fractional order of damping (α), the rotational speed ratio (ϖ) and the mass eccentricity of rotor (e) are investigated, and the sensibility of these faults are also analyzed. Correspondingly, some critical values and ranges are proposed.

The rest of the paper is organized as follows: In Section 2, the preliminaries are presented. The model of GCRSS is established in Section 3. Section 4 discusses the stability of GCRSS with different parameters and Sect. 5 closes the paper.

2. Preliminaries

2.1. Definition of fractional derivative

There are three widely accepted definitions for the fractional derivative, including Grunwald-Letnikov (G-L), Riemann-Liouville (R-L) and Caputo definitions. R-L and Caputo definitions are an improvement over G-L definition. Comparing R-L with Caputo definitions, it is the largest difference that Caputo definitions attach great importance to the selection of initial conditions. Besides, Caputo definitions are more appropriate in science and engineering research [38]. Based on above, Caputo definition [39] will be applied to study the dynamic characteristics of cracked generator rotor system, and it can be written as:

$$ {}_C D_t^q f(t) = \frac{1}{\Gamma(n-q)} \int_a^t (t-u)^{n-q-1} f^{(n)}(u) du, \tag{1}$$

where $n-1 \le q \le n$, $n \in N$; $\Gamma(*)$ is the famous Gamma function; q is the order of the fractional derivative; $f(t)$ is an integrable function.

2.2. Numerical algorithms

Based on the fact that the fractional differential equation can be transformed into Volterra integral equation, many scholars have established an Adams-Bashforth-Moulton predictor-corrector scheme, a time domain approach, to solve the fractional differential equation [40]. This method is more accurate than frequency domain approach, and thus, it will be used in the latter simulation.

Adams-Bashforth-Moulton predictor-corrector scheme can be described as the following fractional differential equation:

$$ \begin{cases} D_t^q y(t) = f(y(t), t) \\ y^{(k)}(0) = y_0^k, \qquad k = 0, 1, \dots, n-1 \end{cases}, \tag{2}$$

which is equivalent to the Volterra integral equation,

$$ y(t) = \sum_{k=0}^{[q]-1} y_0^{(k)} \frac{t^k}{k!} + \frac{1}{\Gamma(q)} \int_0^t (t-u)^{q-1} f(u, y(u)) du. \tag{3}$$

Discretizing Eq. (3), we can get

$$ \begin{aligned} y_h(t_{n+1}) = & \sum_{k=0}^{m-1} y_0^{(k)} \frac{t_{n+1}^k}{k!} + \frac{h^q}{\Gamma(q+2)} f\left(t_{n+1}, y_h^p(t_{n+1})\right) \\ & + \frac{h^q}{\Gamma(q+2)} \sum_{j=0}^{n} a_{j,n+1} f\left(t_j, y_n(t_j)\right), \end{aligned} \tag{4}$$