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Prisoner's dilemma game on reputation-based weighted network

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ABSTRACT

In this paper, we study the prisoner's dilemma game on reputation-based weighted network. We define a player's reputation as the frequency of cooperation in the past few time steps. The edge weight of a link is determined by the reputation of the two players at both ends. The payoff of an agent is multiplied by the value of edge weight. Compared with the unweighted network, the cooperation level is promoted strongly on the reputation-based weighted network. Moreover, we find that the cooperation level declines when the length of credit history becomes longer.

1. Introduction

Understanding the persistence of cooperation among selfish individuals remains a challenge. So far, evolutionary game theory has provided a powerful mathematical framework to address this problem [1]. Researchers have proposed various game models, among which the prisoner's dilemma game (PDG) has been a prevailing paradigm to investigate the emergence of cooperation among individuals through pairwise interactions [2].

Considering the rapid development of complex network theory, much effort has been devoted to the evolutionary game on complex networks in the past decade [3–12]. To explain the emergence of cooperation, researchers have proposed many important mechanisms, such as network reciprocity [13,14], memory effects [15–17], noise [18,19], punishment and reward [20–23] migration [24–26], social diversity [27–30], voluntary participation [31,32] and aspiration [33,34], to name but a few. Besides the above scenarios, weighted network has attracted considerable attention as a more extensive description of structured populations [35–40]. Several typical examples include: Buesser et al. found that the presence of link weights that are correlated in a particular manner with the degree of the link endpoints, leads to unprecedented level of cooperation [36]. Ma et al. arranged three types of weight distributions: exponential, power-law and uniform distributions, and the weight is assigned to links between players. They found that the power-law distribution enables the highest promotion of cooperation and the uniform one leads to the lowest enhancement, whereas the exponential one lies often between them [37]. Additionally, Du et al. revealed that cooperative behavior can be more facilitated when edge weights are heterogeneous rather than homogeneous [38].

In the above mentioned works, the weights of links are regarded as fixed and invariable. But in the real-world systems, the relation strengths between individuals are constantly changed for various reasons, such as reputation. The reputation mechanism is considered as an effective approach to help the cooperators to resist the invasion of defectors. Based on reputation, individuals choose interaction partners [41,42], decide whether to participate in the game [43,44] or not, and decide whether to cooperate or to defect [45]. In this paper, we assume that the edge weight is determined by the reputation of the two connected players. Here a player's reputation is defined as the frequency of cooperation in the past few time steps.

The following of this paper is organized as follows. In Section 2, we introduce the prisoner's dilemma games model and the reputation-based weight mechanism. In Section 3, we show the experimental results and discussions. In Section 4, we summarize our findings.

2. Models and Methods

In the original PDG played by two agents, players get reward R or punishment P if both cooperate or defect. If one cooperator meets one defector, the former gains the sucker's payoff S while

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the latter obtains the temptation T, these payoffs satisfy the ranking $T > R > P > S$. We simplify the payoff matrix in accordance with common practice: $R = 1$, $T = b > 1$, and $P = S = 0$ [46]. As a result, intelligent players will be in a dilemma. That is, defection is the best choice for one player but the population cannot maintain if all players select defection.

The two strategies: cooperation or defection, are described by

$$s_i(t) = \begin{pmatrix} 1 \\ 0 \end{pmatrix} or \begin{pmatrix} 0 \\ 1 \end{pmatrix} \tag{1}$$

respectively. At time t, a player i gets its accumulated payoff as follows

$$U_i(t) = \sum_{j \in \Omega_i} w_{ij}(t) s_i(t)^T A s_j(t), \tag{2}$$

where Ω_i is the set of neighbors of player i, $w_{ij}(t)$ is the weight of the edge that connects players i and j, and A is the rescaled payoff matrix given by

$$A = \begin{pmatrix} 1 & 0 \\ b & 0 \end{pmatrix}. \tag{3}$$

In our study, the weight of an edge is symmetry, that is, $w_{ij} = w_{ji}$. Without losing generality, the weight of each edge is set to be unit initially. With time the edge weight is changed according to the reputation. The reputation of a player i at time t is defined as the frequency of cooperation in the last L time steps [43], that is

$$R_i(t) = \frac{\sum_{m=1}^{L} s_i(t-m)}{L}. \tag{4}$$

The edge weight $w_{ij}(t)$ of a link is determined by the reputation of the two players at both ends i and j, that is

$$w_{ij}(t) = w_{ji}(t) = R_i(t) + R_j(t). \tag{5}$$

The node weight of agent i at time t is defined as

$$D_i(t) = \sum_{l \in \Omega_i} w_{il}(t), \tag{6}$$

where l runs over all the neighbors of agent i.

After accumulating the payoffs, all players synchronously update their strategies. Whenever a site i is updated, it will randomly pick up a neighbor j and by adopting $j's$ strategy with the probability given by the Fermi function

$$W_{i \to j} = \frac{1}{1 + e^{[(U_i(t) - U_j(t))/K]}}, \tag{7}$$

where $U_i(t)$ and $U_j(t)$ are the payoffs of players i and j, respectively. The parameter $K = 0.1$ characterizes the environmental noise during the strategy adoption, reflecting irrationality of individuals and errors [47,48].

3. Results and discussions

In the following studies, we consider the evolutionary PDG on a 200*200 square lattice with periodic boundary condition, but the qualitative results remain valid also if we use larger lattices. Initially each player is designated either as a cooperator or defector with equal probability. The key quantity that characterizes the cooperative behavior of a system is the cooperation fraction ρ_c when the system reaches dynamical equilibrium. In all of the following simulations, the data are obtained by averaging over the last 5000 generations of the entire 100000 generations. Each piece of data is an average of 50 individual runs.

Figure 1 shows the average level of cooperation ρ_c as a function of the temptation to defect b for different values of the memory length L. One can see that for each value of L, ρ_c decreases as b increases. For the traditional case (the weight for each edge is fixed to be 1), ρ_c cannot reach 1 even $b = 1$. However, if we take the reputation-based weight into account, the cooperation level is enhanced efficiently and the full cooperation can be achieved when b is small. Of particular note is that the cooperation level is obviously affected by the parameter L. As shown in Fig. 1, $L = 1$ is optimal for the evolution of cooperation. Moreover, the system exhibits the typical phase transition process regarding the temptation to defect b, in which there exist two critical thresholds named after the lower threshold (b_{c1}) and upper threshold (b_{c2}), respectively. The full cooperation (defection) arises when b is less (more) than b_{c1} (b_{c2}). Cooperators and defectors coexist when b lies between b_{c1} and b_{c2}. We plot the full $b - L$ phase diagram in Fig. 2. It can be clearly shown that the critical thresholds (b_{c1} and b_{c2}) become smaller with the increase of L. Since that b_{c1} declines greater than b_{c2}, the region of $C + D$ phase will be enlarged as the memory length L increases.

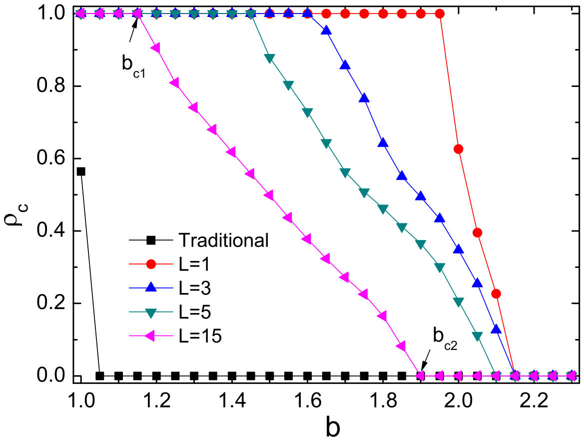

Fig. 1. (Color online) The fraction of cooperators ρ_c as a function of the temptation to defect b for different values of the memory length L. It can be seen that compared with the traditional situation (i.e. the weight for each edge is fixed to be 1), the introduction of reputation-based weight can promote cooperation greatly. Moreover, $L = 1$ is found to be optimal to enhance cooperation. For $L > 0$, there exist two critical thresholds b_{c1} and b_{c2}, corresponding to the extinction of defectors and cooperators, respectively.

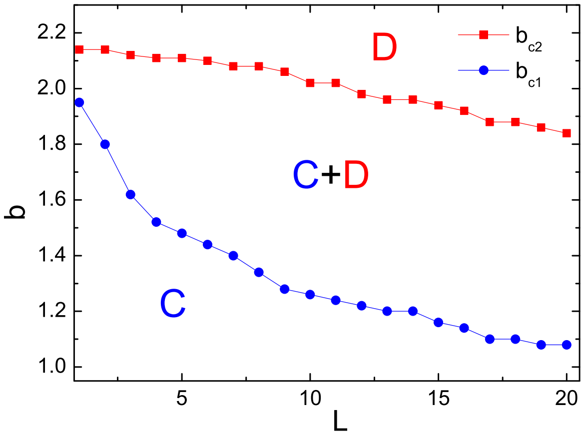

Fig. 2. (Color online) Full $b - L$ phase diagram. There are three phases: full cooperators (C), full defectors (D), and the coexistence of cooperators and defectors ($C + D$). As the memory length L increases, the critical thresholds corresponding to the full cooperation (b_{c1}) and the full defection (b_{c2}) decrease. Moreover, the region for $C + D$ phase becomes wider as L increases.

In Fig. 3 we plot the cooperator frequency ρ_c as a function of the memory length L for different values of the temptation to defect b. For each value of b, ρ_c reaches the highest when $L = 1$, indicating that cooperation is best promoted when only the latest