Chaos, Solitons and Fractals 110 (2018) 69–75

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Fractality in market risk structure: Dow Jones Industrial components case

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ARTICLE INFO

Article history:
Received 3 January 2018
Accepted 21 February 2018

Keywords:
Capital asset pricing model
Scaling
Detrended cross-correlation analysis
Detrending moving-average cross-correlation analysis

ABSTRACT

We examine the Dow Jones Industrial Average index components with respect to the capital asset pricing model (CAPM), specifically its scaling properties in the sense of different investment horizons. To do so, we use the novel methods of fractal regressions based on the detrended cross-correlation analysis and the detrending moving-average cross-correlation analysis. We report three standard groups of stocks – aggressive, defensive and market-following – which are rather uniformly represented. For most of the stocks, the β parameter of the CAPM does not vary significantly across scales. There are two groups of exceptions. One of aggressive stocks which are even more aggressive for short investment horizons. These do not provide portfolio diversification benefits but allow for high profits above the market returns and even more so for the short investment horizons. And the other group of more defensive stocks which become very defensive in the long term. These stocks do not deliver short term profits but can serve as strong risk diversifiers. Apart from these direct results, our analysis opens several interesting questions and future research directions, both technical and experimental, which we discuss in more detail.

1. Introduction

The Global Financial Crisis and its aftermath have shown that the notion of systematic (and also systemic) risk is not vain. During the critical periods, practically all stocks kept losing their value, and the losses and risk could not have been diversified away. Dating back to Markowitz [1], diversification, i.e. lowering the portfolio risk by its enlarging, is tightly connected to the correlation structure of the market. If the assets are all strongly correlated, they will rise and fall together. Only a single asset moving against the market can lower the portfolio risk markedly. Connection between the market risk and individual assets' risk is nicely captured by the capital asset pricing model (CAPM), which has become one of cornerstones of the modern financial economics since its introduction in the 1960s [2–4]. The model describes the relationship between an asset and market in a simple linear manner. Regardless its simplicity, the model has several intuitive but important implications. The most important one from the portfolio construction perspective is the existence of the market (systematic) risk that cannot be diversified away. In words, as most assets are at least somehow connected to the global market movements, this principal component cannot be gotten rid of as it is common to all said assets. Another appealing outcome of the model's simplicity is that it is described by only two parameters one of which – β – identifies the asset as an aggressive one, a defensive one, or a market-following one. However, if the past decade has taught the financial theorist and practitioners anything, market participants can perceive an asset behavior differently. There are different types of investors with different trading strategies and different investment horizons and it is hard to believe they all agree on risk specifics of a given asset as called for by the efficient market hypothesis [5,6]. Quite the contrary, it is more realistic to assume that the market participants differ as well as their expectations as asserted by the fractal market hypothesis [7,8]. Our main motivation is thus to inspect the stock markets via the capital asset pricing model with a special focus on scale specifics of the model. To do so, we utilize the quite newly proposed regression frameworks build on the fractal methods, specifically the detrended cross-correlation analysis and the detrending moving-average cross-correlation analysis. In addition, we provide a novel approach towards the statistical significance of the scale variability.

The paper is organized as follows. The next section describes the capital asset pricing model in detail and focuses on the fractal methods and how to approach statistical inference in the CAPM setting. The following section introduces the analyzed data and explains the specific choices. The last section presents the results, provides economic interpretation and sketches some further venues into the topic.

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https://doi.org/10.1016/j.chaos.2018.02.028
0960-0779/

2. Methods

2.1. Capital asset pricing model

The capital asset pricing model (CAPM) is one of the important building blocks of the modern financial economics as it describes the relationship between risk and return in the rational equilibrium market. Building on the Markowitz modern portfolio theory [1,9], the CAPM was developed by several authors independently of one another [2–4]. For individual assets, the model is stated as

$$\mathbb{E}(R_i) = R_f + \beta_i(\mathbb{E}(R_m) - R_f) \tag{1}$$

where $\mathbb{E}(R_i)$ is the expected return of asset i, R_f is the risk-free rate, and $\mathbb{E}(R_m)$ is the expected market return. β_i is the crucial parameter of the model and it can be interpreted as a sensitivity of the asset return to the market return (both cleared by the risk-free rate). With respect to Refs. [2–4], it can be shown that

$$\beta_i = \rho_{im}\frac{\sigma_i}{\sigma_m} = \frac{\mathrm{cov}(R_i, R_m)}{\mathrm{var}(R_m)} \tag{2}$$

where ρ_{im} is the correlation between R_i and R_m, σ_i is the standard deviation of R_i, and σ_m is the standard deviation of R_m. Note that the representation of β_i on the right-hand side of Eq. (2) is the same as the least squares estimator of the simple regression

$$R_i - R_f = \alpha_i + \beta_i(R_m - R_f) + u_i \tag{3}$$

where u_i is the error term and α_i is a deviation from the equilibrium return.[1] The β parameter can be thus easily estimated using the least squares methodology. In general, β can attain any value but cases when $\beta \leq 0$ are rare (assets moving against the market, or short positions). Apart from this unlikely case, there are three interesting cases:

- $0 < \beta < 1$: defensive assets, which move in the same direction as the market but have lower volatility
- $\beta = 1$: assets following the market, e.g. market-index-based assets, or strong contributors to a market index
- $\beta > 1$: aggressive assets, which move in the same direction as the market but with higher volatility

The definition of β and the CAPM construction imply that the market return R_m and error term u in Eq. (3) are uncorrelated. This allows to split the asset variance (risk) σ_i^2 into two orthogonal components as

$$\sigma_i^2 = \beta_i^2\sigma_m^2 + \sigma_{u_i}^2 \tag{4}$$

where σ_m^2 is the market variance and $\sigma_{u_i}^2$ is the error term variance [10]. The component $\beta_i^2\sigma_m^2$ is called the systematic risk associated with the market and it cannot be eliminated (i.e. by diversification). The error term variance is usually referred to as the idiosyncratic risk (or specific risk or unsystematic risk) and this one can be eliminated (or at least mitigated). High β assets can thus return high profits in the growing market but they do not contribute to risk optimization. Therefore, a high β portfolio is possibly very profitable but also very risky. Low β assets thus help diversifying the risk.

The capital asset pricing model is connected to an understanding of a market as an efficient one with respect to the efficient market hypothesis (EMH) [5,6,11–14], specifically to one of its assumptions that the investors are homogeneous in their expectations and have a common investment horizon [15]. However, observing reality suggests that investors are far from homogeneous and they differ in their investment horizons, ranging from algorithmic and noise trading (with very short horizons in a span of second fractions) to pension funds (with long investment horizons of several years or even decades). Specifically, we want to examine whether an asset can be seen in a different perspective (in the CAPM sense) by a short-term investor and a long-term investor, i.e. whether the asset βs can be different for different investment horizons. For this purpose, we utilize the regression frameworks build with scaling and fractality in mind – fractal regressions based on the detrended cross-correlation analysis and the detrending moving-average cross-correlation analysis.

2.2. Fractal regressions

The capital asset pricing model is based on a bivariate relationship between an asset's return (corrected by the risk-free rate) an a market return (also corrected by the risk-free rate). The model can be thus ideally studied by quite recently proposed regression frameworks based on the detrended fluctuation analysis (DFA) and detrending moving average (DMA) procedures [16,17]. Here, the methods are not only useful due to their robustness to persistence, short-range correlations and heavy tails [18–20], but specifically for their ability to study the relationship between series at different scales so that we can distinguish between short-term and long-term investment horizons. This leads to possible findings such that a specific stock is considered to be an aggressive investment for short-term investors but a defensive (safe) investment for long-term investors. Such results would support claims of the fractal markets hypothesis (FMH) [7,8,21–23] as opposed to the efficient market hypothesis (EMH) [5,6,11–14], which assumes that all investors agree on the riskiness of a specific asset.

The two fractal regression frameworks are based on the methods usually used for detecting fractal structure and long-range dependence properties of analyzed series – specifically the detrended fluctuation analysis (DFA) [24] and the detrending moving average (DMA) [25,26]. Both methods have been generalized for analysis of bivariate properties of the series which has given rise to the detrended cross-correlation analysis (DCCA) [27–29] and the detrending moving-average cross-correlation analysis (DMCA) [30,31]. Combination of DFA and DCCA allowed for an introduction of the DCCA-based correlation coefficient which describes correlations between series at different scales [32]. In the same logic, DMA and DMCA have been combined to form the DMCA-based correlation coefficient [33], which surpasses the original DCCA-based method under various specifications of long-range dependence [34]. These scale-specific correlation coefficients have been extensively used in empirical studies across disciplines [28,35–46]. The regression frameworks are only a step away from the correlation analysis.

The DCCA and DMCA-based correlation coefficients are based on a simple idea of substituting the covariance and variances (standard deviations) in the definition of correlation coefficient with the scale-specific covariances and variances obtained during the DFA/DCCA and DMA/DMCA procedures. Without a need to eventually arrive at the Hurst exponent given by DFA and DMA, we can use the fluctuation functions obtained during the procedures. Specifically for the DFA procedure, we select the scale s and split the profile series (integrated demeaned original series) into boxes of given length.[2] In each box, a polynomial trend (usually linear as in our application) is fitted, residuals are obtained and mean squared error is calculated. The mean squared errors are then averaged over all boxes of size s and to get $F^2_{X,DFA}(s)$. For the bivariate

[1] The α parameter can be used for investment decisions as $\alpha > 0$ suggests overpricing of the asset and $\alpha < 0$ suggest underpricing of the asset. However, we focus primarily on the β parameter here and leave possible α discussions for future research.

[2] If the series is not divisible by s, we divide the series into boxes from the beginning and from the end, i.e. obtained twice as many boxes compared to the divisible case.