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Synchronization of fractional-order memristor-based complex-valued neural networks with uncertain parameters and time delays

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ABSTRACT

This paper talks about the global asymptotical synchronization problem of delayed fractional-order memristor-based complex-valued neural networks with uncertain parameters. Under the framework of Filippov solution and differential inclusion theory, several sufficient criteria ensuring the global asymptotical synchronization for the addressed drive-response models are derived, by means of Lyapunov direct method and comparison theorem. In addition, two numerical examples are designed to verify the correctness and effectiveness of the theoretical results.

1. Introduction

In the past few years, complex-valued neural networks (CVNNs) have been applied to extensive practical applications, especially, in the fields of dealing with electromagnetic, filtering, ultrasonic, image processing, quantum waves, optoelectronics, speech synthesis, and other areas. As is well known, when compared with the real-valued neural networks (RVNNs), the states, connection weights and activation functions of CVNNs are all depicted in terms of complex-valued data, and the information processing process of CVNNs is realized on the complex plane [1]. Thus, CVNNs have much more complicated properties and can explore new capabilities and higher performance, which make it possible to dispose problems that cannot be solved with the real-valued counterparts. For example, a single complex-valued neuron with the orthogonal decision boundaries can be used to resolve such problems as the XOR problem and the detection of symmetry problem, while a single real-valued neuron could not be managed, which reveals the potent computational capabilities of complex-valued neurons [2,3]. Hence, it is of great importance to tackle the dynamics of CVNNs deeply. In recent years, numerous results have been reported on the dynamical behaviors of CVNNs, see [4–13] and references therein.

The concept of memristor which describes the relationship between electric charge and magnetic flu was originally theorized by Chua [14] in 1971, and it was predicted as the fourth passive circuit element. In 2008, the prototype of practical memristor device was realized by scientists at the Hewlett-Packard Labs [15]. It is found that the memristor, a two-terminal element with variable resistance called memristance, has idiosyncratic circuit properties compared with the other circuit elements such as resistors, capacitors and inductors. The major superiority of the memristor is that the value of resistance depends on the magnitude and polarity of the voltage implemented in it and the time duration that the voltage has been applied, and also have the potentiality to remember the most recent resistance when the implemented voltage is turned off [16,17]. On the strength of these features, more and more researchers begin to pay their attention and increasing interest to analysis the properties of memristors, some of which have incorporated the memristor into the integrated circuit design of neural networks named as memristor-based neural networks (MNNs), in which the resistor elements in the circuit have been replaced by memristors. Recently, the analysis of dynamics of MNNs has been a hot topic, and a considerable of excellent results have been reported in the literature, see [18–26] and references therein.

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Fractional calculus, a generalization of ordinary differentiation and integration, did not attract much attention for ages due to lack of application background. Nowadays, fractional calculus has become an active research field since some researchers point out that many phenomena in various fields of science and engineering could be described by fractional calculus and fractional differential equations, such as viscoelastic systems, stochastic diffusion, dielectric polarization, quantum mechanics, molecular spectroscopy, electrode-electrolyte polarization and electromagnetic waves. This mainly benefits from the fact that fractional-order derivatives provide an excellent instrument for the description of memory and hereditary properties of various materials and processes [27,28]. In recent years, fractional derivative and integral have been integrated into neural network models to form fractional-order neural network models. It is worth mentioning that the incorporation of memory terms into neural networks is an extremely important improvement. Thereupon, the analysis of dynamics on fractional-order neural networks (FNNs) has been become an increasing interest and growing area of research from then on, see [29–52,52] and references therein. Among which, the synchronization analysis of fractional-order neural networks has been captured wide attention. In [29,30], projective synchronization and adaptive synchronization of FMNNs were discussed. In [31], by employing linear delay feedback control, synchronization and anti-synchronization problems of fractional-order complex-valued neural networks (FCVNNs) were concerned. In [32,34], based on the concept of Filippov solution for FNNs in the sense of Caputos fractional derivation, authors investigated global Mittag–Leffler synchronization of fractional-order memristor-based neural networks (FMNNs) and global Mittag–Leffler synchronization of FNNs with discontinuous activations, respectively. In [41], by applying Laplace transform and the generalized Gronwalls inequality, several sufficient criteria were deduced to ensure the finite-time synchronization for FMNNs.

As is well-known, the exact values of parameters of the addressed models are usually cannot obtained, the major reason for this is that the perturbations in the models and the disturbances in the environment always exist, which would cause parameter uncertainties. Effects of such parameter uncertainties should not be ignored in the analysis of dynamical behaviors of nonlinear systems since they may derail the stability, synchronization or some other properties. Recently, many research literatures have focused on the synchronization of neural network model with parameter uncertainties, see [53–58]. In [53–57], authors incorporated the uncertain parameters into the addressed models, and investigated the robust fixed-time synchronization of inter-order CVNNs, the adaptive exponential synchronization of integer-order Cohen-Grossberg CVNNs, robust synchronization of integer-order MNNs and lag stochastic synchronization of integer-order NNs, respectively. In [58], synchronization of delayed FMNNs with unknown parameters were concerned. To our best knowledge, the parameters of most previous literatures of fractional-order memristor-based complex-valued neural networks (FMCVNNs) are deterministic. Few works have been done on the synchronization of FMCVNNs with parameter uncertainties.

Strongly motivated by the aforementioned discussions, the main objective of this paper is to investigate the problem of synchronization of delayed FMCVNNs with unknown parameters. Based on the fractional differential equations, complex-valued network theory and differential inclusion theory, the drive-response models of FMCVNNs with delays and parameter uncertainties will be established. Afterwards, by employing feedback control strategy and Lyapunov direct method, several sufficient criteria ensuring the global asymptotical synchronization for the concerned network models will be derived. Finally, numerical examples will be designed to verify the availability and feasibility of the theoretical results. The main contribution of this paper can be summarized in brief as follows: (1) the parametric uncertainties are brought into FMCVNNs. (2) the synchronization analysis of FMCVNN is presented, and based on differential inclusion theory and Lyapunov direct method, some criteria ensuring the synchronization of the addressed drive-response models are established. (3) compared with the exist results in the literature, the obtained main results are more general and less conservative.

2. Model description and preliminaries

Definition 1 [27]**.** The Caputo fractional derivative of order α for a function $z(t)\in C^{n+1}[[0,+\infty),\mathbb{R}]$ (the set of all n-order continuous differentiable functions on $[0,+\infty)$) is defined as

$$D^{\alpha}z(t)=\frac{1}{\Gamma(n-\alpha)}\int_0^t(t-s)^{n-\alpha-1}z^{(n)}(s)ds,$$

where $\alpha>0$, $\Gamma(\alpha)=\int_0^{+\infty}e^{-t}t^{\alpha-1}dt$, and n is the first integer greater than α, that is, $n-1<\alpha<n$, and in particular, when $0<\alpha<1$, one has

$$D^{\alpha}z(t)=\frac{1}{\Gamma(1-\alpha)}\int_0^t(t-s)^{-\alpha}z'(s)ds.$$

Consider the following delayed fractional-order memristor-based complex-valued neural networks with unknown parameters

$$\begin{aligned}D^{\alpha}z_j(t)&=-c_jz_j(t)+\sum_{k=1}^{n}[a_{jk}(z_k(t))+\Delta a_{jk}(t)]f_k(z_k(t))\\&\quad+\sum_{k=1}^{n}[b_{jk}(z_k(t))+\Delta b_{jk}(t)]g_k(z_k(t-\tau))+J_j(t),\end{aligned}\quad(1)$$

where $\alpha\in(0,\ 1)$, $t\ge 0$. $z(t)=(z_1(t),z_2(t),\dots,z_n(t))^T$, $z_j(t)$ represents the complex-valued state variable associated with the jth neuron; $c_j>0$ stands for the decay rate coefficient; $f_k(z_k(t))$ and $g_k(z_k(t-\tau))$ stand for the complex-valued activation functions of the kth unit at time t and $t-\tau$, respectively; $a_{jk}(z_k(t))$ and $b_{jk}(z_k(t))$ denote, respectively, the synaptic connection weight of the kth unit to the jth unit at time t and $t-\tau$; $\Delta a_{jk}(t)$ and $\Delta b_{jk}(t)$ mean the deviation of $a_{jk}(z_k(t))$ and $b_{jk}(z_k(t))$, respectively; $J_j(t)$ denotes the bounded external input, and the memristor-based connection weights $a_{jk}(z_k(t))$ and $b_{jk}(z_k(t))$ satisfy

$$a_{jk}(z_k(t))=\frac{\mathbb{W}_{jk}}{C_j}\times\mathrm{sgn}_{jk},\quad b_{jk}(z_k(t))=\frac{\mathbb{M}_{jk}}{C_j}\times\mathrm{sgn}_{jk},$$

$$\mathrm{sgn}_{jk}=\begin{cases}1, & j\neq k,\\ -1, & j=k,\end{cases}$$

in which $\mathbb{W}_{jk}$ and $\mathbb{M}_{jk}$ are the memductances of resistors $\mathbb{R}_{jk}$ and $\mathbb{F}_{jk}$, $\mathbb{R}_{jk}$ is the resistor between the feedback function $f_j(z_j(t))$ and $z_j(t)$, and $\mathbb{F}_{jk}$ is the resistor between the feedback function $g_j(z_j(t-\tau))$ and $z_j(t)$.

The initial conditions associated with (1) are

$$z_j(h)=\rho_j(h)+i\delta_j(h),\quad h\in[-\tau,0].\quad(2)$$

In this paper, we will handle at discussing the drive-response synchronization, and the response system of (1) can be depicted as

$$\begin{aligned}D^{\alpha}s_j(t)&=-c_js_j(t)+\sum_{k=1}^{n}[a_{jk}(s_k(t))+\Delta\tilde{a}_{jk}(t)]f_k(s_k(t))\\&\quad+\sum_{k=1}^{n}[b_{jk}(s_k(t))+\Delta\tilde{b}_{jk}(t)]g_k(s_k(t-\tau))\\&\quad+J_j(t)+W_j(t),\end{aligned}\quad(3)$$