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Synchronization dependence on initial setting of chaotic systems without equilibria

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ABSTRACT

An initial-dependent system is developed from an electromotor driven by nonlinear torque. Standard nonlinear analysis is carried out and chaotic region is explored in the dynamical system without equilibria. Phase portrait, Lyapunov exponent spectrum, Hamilton energy and bifurcation analysis are calculated to confirm the emergence of chaos and state selection. It is found that the attractor type (periodical or chaotic) is dependent on the initial setting. Furthermore, bidirectional coupling is used to detect the synchronization approach between two initial-dependent electomotors. In the case of network synchronization and pattern selection, a chain network is designed and statistical factor of synchronization is calculated to predict the synchronization stability on the network. It is found that the synchronization stability shows some dependence on initial setting for one variable(external load). The Hamilton energy is also calculated to find the behavior dependence on initial setting and parameter selection by using Helmholtz theorem.

1. Introduction

The emergence of chaos and hyperchaos throws important insights to understand the intrinsic nonlinear characteristic in dynamical systems by using nonlinear analysis [1–5]. Chaos can be found in physical, biological, chemical systems and also engineering system as well. Based on some mathematical models [6–10], the dynamical behaviors for chaos can be reproduced by estimating the strange attractors, Lyapunov dimension, power spectrum etc. One important application for chaos is that chaotic system can be used for image encryption and secure communication by triggering more reliable secure keys [11–15]. As is well known, the dynamics of nonlinear systems is much dependent on the parameter region and non-linearity which bridges the relation between different variables. By enhancing the nonlinear terms in the dynamical systems, multi-attractors can be induced when more equilibrium points are formed [16–18].

In the last decades, chaos synchronization and control have been investigated extensively [19–25]. Considering the control cost and reliability for controllers, smaller power consumption [26,27] and shorter transient period reaching the target orbits are much appreciated. Therefore, adaptive-control schemes are often used to control chaotic systems and realize synchronization even some parameters are unknown. That is to say, the unknown parameters can be estimated exactly when complete synchronization is reached [28–30]. Both rotating motors and moving objects, enough energy supply is critical to keep the motion. Therefore, it is important to discuss the dynamical behavior dependence on energy supply when these dynamical systems are described by dimensionless dynamical models. Based on the Helmholtz theorem [31], Hamilton energy [32–35] is also calculated to find the mode dependence on bifurcation parameters and nonlinear terms, and it is confirmed that modulation on Hamilton energy can induce mode transition in periodical oscillators. As is well known, the output series and orbits of chaotic system are dependent on the initial values from the beginning, while the phase portraits and attractors become invariable when all the parameters are fixed. For example, the excitability of neuron (or some excitable media) is dependent on the intrinsic parameter setting, but external stimulus can also be effective to modulate the excitability as well. For example, external stimulus with mixed frequency and electromagnetic radiation can induce distinct mode transition in electrical activities [36,37].

However, memristive electric device such as memristor [38–43] can adjust the non-linearity in the chaotic circuits and thus the dynamical behaviors can be controlled, the potential mechanism is that the memductance of memristor (bifurcation

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parameter) is much dependent on the external input current completely. As a result, Ma et al. [44] proposed a generic dynamical system and explained the potential mechanism for state dependence on initial values setting for the dynamical systems, which resetting for initial values will trigger switch between periodical and chaotic states even parameters are fixed. That is, resetting parameters in the dynamical system will change the dynamical behaviors and also the synchronization approached is controlled by the intrinsic parameters and coupling intensity between nodes (or neurons) [45]. Besides the potential application on designing nonlinear circuits, memristor is often used to bridge the coupling between magnetic flux and membrane potential of neurons with electromagnetic induction is under consideration [46–50]. Based on these improved neuron and cardiac tissue model, the effect of electromagnetic radiation on mode transition [46,47] in electrical activities can be observed, and heart disease[49] induced by electromagnetic radiation can be explained in feasible way. Furthermore, the interesting synchronization phenomenon under electromagnetic radiation can be detected and explained [50].

In this paper, a three-dimensional autonomous system without equilibrium is proposed when nonlinear damping is applied on an electromotor. The dynamical characterization is investigated by using nonlinear analysis, then synchronization stability between two coupled motors with loads, and network synchronization and pattern selection are discussed when each motor is coupled by two nearest-neighbor motors on the chain network.

2. Model and dynamical analysis

According to the physical rotation theorem, a rotor can be accelerated by applying external torque as follows

$$J\frac{d^2\theta}{dt^2} = M \tag{1}$$

where θ, J, M represents the angular displacement, moment of inertia and external torque, respectively. For an electromotor, J is used to calculate the moment of inertia between motor load and the application of gear drive. For realistic motors, the damping torque can slow down the rotation in nonlinear way. In case of low speed rotation, the damping torque is described by

$$M_{block} = -K\frac{d\theta}{dt} \tag{2}$$

where K defines the damping intensity for the rotating motor with load, then the dynamical equation for electromotor with load can be calculated by

$$J\frac{d^2\theta}{dt^2} + K\frac{d\theta}{dt} = M \tag{3}$$

For simplicity, the moment of inertia is set as $J=1$, and the angular variable θ is replaced by new variable x as well. As a result, the dynamical equations can be rewritten as follows

$$\begin{cases} \dfrac{dx}{dt} = y \\ \dfrac{dy}{dt} = -x - Ky \end{cases} \tag{4}$$

where the variable y denotes angular velocity for the motor, it suggests that the damping intensity is dependent on the angular velocity in nonlinear type, which is defined as follows

$$K(z) = k_0 z^2 - z; \quad \frac{dz}{dt} = a - y^2 \tag{5}$$

where variable z can be thought as external load imposed on the electromotor thus the intensity of damping is dependent on the external load. As a result, a three-variable dynamical system with quadratic non-linearity is approached by

$$\begin{cases} \dfrac{dx}{dt} = f_1(y) = y \\ \dfrac{dy}{dt} = f_2(x, y, z) = -x + yz - k_0 z^2 y \\ \dfrac{dz}{dt} = f_3(y) = a - y^2 \end{cases} \tag{6}$$

where a, k_0 are parameters for Eq. (6), and both of the parameters describe the damping effect. In the dynamical view, appropriate setting for the two parameters can trigger periodical and even chaotic states in this nonlinear system. Inspired by the works in Ref. [44], the Eq. (6) becomes initial-dependent because slight difference in initial setting for variable z can induce transition of orbits. The potential mechanism could be that $k_0 z^2$ can trigger different feedback for variable y at the beginning. It is confirmed that invariance of system occurs by applying coordinate transformation as $(x, y, z) \to (-x, -y, z)$. Therefore, the attractor can show some symmetry along the third variable z. Furthermore, the dissipativitiy on Eq. (6) can be estimated by

$$\nabla V(x, y, z) = \frac{\partial f_1}{\partial x} + \frac{\partial f_2}{\partial y} + \frac{\partial f_3}{\partial z} = z - k_0 z^2 \tag{7}$$

As a result, the dynamical system shown in Eq. (6) become convergent and asymptotically stable at $z < k_0 z^2$. The dynamics is associated with the equilibrium, which solution for equilibrium points can be approached by

$$\begin{cases} y = 0 \\ -x + yz - k_0 z^2 y = 0 \\ a - y^2 = 0 \end{cases} \tag{8}$$

It is confirmed that (0, 0, 0) is one equilibrium point at $a=0$. Otherwise, no equilibrium points can be detected at $a \neq 0$. Based on the Helmholtz theorem [51], the dynamical equations shown in Eq. (6) can be rewritten in vector field as follows

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = f_c(x, y, z) + f_d(x, y, z) = \begin{pmatrix} y + ak_0 z^2 \\ -x - k_0 z^2 y - 1 \\ a - y^2 + ax \end{pmatrix} + \begin{pmatrix} -ak_0 z^2 \\ 1 + zy \\ -ax \end{pmatrix} \tag{9}$$

The Hamilton energy $H(x, y, z)$ [34] can be estimated by

$$\begin{cases} \nabla H^T f_c(X) = 0, \\ \nabla H^T f_d(X) = \dot{H} = dH/dt \end{cases} \tag{10}$$

The solution for Eq. (10) can be approached by

$$H = \frac{1}{2}x^2 + x + \frac{1}{2}y^2 - \frac{k_0}{3}z^3 \tag{11}$$

Indeed, the Hamilton energy is dependent on the damping factor k_0. Before further dynamical analysis, the synchronization between two electromotors is described by

$$\begin{cases} \dot{x} = y + g(\hat{x} - x) \\ \dot{y} = -x + yz - k_0 z^2 y \\ \dot{z} = a - y^2 \\ \dot{\hat{x}} = \hat{y} + g(x - \hat{x}) \\ \dot{\hat{y}} = -\hat{x} + \hat{y}\hat{z} - k_0 \hat{z}^2 \hat{y} \\ \dot{\hat{z}} = a - \hat{y}^2 \end{cases} \tag{12}$$

where g is the coupling intensity between motors, and bidirectional coupling type is considered. To measure the synchronization