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Walking dynamics of the passive compass-gait model under OGY-based state-feedback control: Rise of the Neimark–Sacker bifurcation

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ABSTRACT

This paper continues our research work on analysis of passive dynamic walking of the two-degree-of-freedom planar compass-gait biped robot under the OGY-based state-feedback control. We showed, in our previous works, that the walking dynamics of the compass-gait model under control exhibits chaos and periodic-doubling and cyclic-fold bifurcations. In a first part, our analysis of the walking behavior was achieved using the impulsive hybrid continuous-time nonlinear dynamics of the compass-gait model under the OGY-based control. In a second part, our study of the controlled gait and the displayed local bifurcations was realized via the controlled hybrid Poincaré map. In the present work, we demonstrate, for the first time, the appearance of the Neimark–Sacker bifurcation in the controlled dynamic walking of the compass-gait model. Our investigation is achieved via the controlled hybrid Poincaré map instead of the impulsive continuous dynamics model of the bipedal walking. For such study, we mainly use bifurcation diagrams and 2D phase portraits of the discrete Poincaré map. We show that such Neimark–Sacker bifurcation is generated from a period-1 gait and is localized in a small range of the bifurcation parameter, the slope angle. We introduce a two-parameter bifurcation diagram to study occurrence of the Neimark–Sacker bifurcation.

1. Introduction

It is well-known that a mechanically suitably designed biped robot can walk down a slope indefinitely without any actuation and with steady periodic and chaotic gaits [1,2]. It was shown that the passive dynamic walking of such biped robot is expected to considerably increase energy efficiency of the bipedal locomotion. Thus, investigating such natural motions of the passive bipedal walking may lead us to develop some methodologies for controlling active walking machines as well as to understand human locomotion [3]. Since the work of McGeer in 1990 [1], analysis of the passive dynamic walking of biped robots has grown exponentially, e.g. [4–14], just to mention a few. We refer also our reader to [15,16] for a literature review.

The walking model of biped robots is described by an impulsive hybrid nonlinear dynamics. More particularly, the dynamics of the simplest compass-gait biped model is composed by a nonlinear differential equation modeling the swing phase and a nonlinear algebraic equation modeling the impulsive effect during the impact phase [2]. Research works conducted under the passive dynamic walking of biped robots are primarily an analysis of its properties, namely dynamics, stability, limit cycle, etc. Stability analysis of the dynamic walking of biped robots was based on the Poincaré map and its linearization. Generally, it is difficult to represent analytically the Poincaré map since it relies on finding the analytical solution to the differential-algebraic equations that describe the motion of the biped robot. Then, the only way to analyze walking dynamics is based on numerical computation of the linearized Poincaré map around some fixed point of the Poincaré map, and then to estimate eigenvalues in order to investigate stability [13]. Coupled to the virtual holonomic constraint, the transverse linearization technique has proved to be an interesting and a powerful method for orbital stabilization around a desired cyclic trajectory of the biped robot [17–19]. Similarly, the orbital stability analysis of bipedal walking dynamics was addressed in [20–22] using the moving Poincaré section approach, which involves a family of transversal surfaces at each point on the periodic trajectory.

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It is known that passive dynamic walking of biped robots undergoes chaos, period-doubling bifurcations, cyclic-fold bifurcations and other complex behaviors as some bifurcation parameter is varied [13,14,23–27]. For the compass-gait model, only the period-doubling bifurcation and the cyclic-fold bifurcation have been identified. These two types of bifurcation are also a common mechanism in the walking dynamics of other biped robots. However, the Neimark–Sacker bifurcation was identified for the walking dynamics of some biped robots with more complex structure than that of the compass-gait biped. Farshimi and Naraghi [28] showed the existence of a certain different route to chaos in a passive biped model with an upper body attached to the legs with a linear torsional spring. Such route was displayed from a Neimark–Sacker-2 bifurcation, which is born from a period-2 stable gait. Moreover, Deng et al. [29] found a variety of gait bifurcation phenomena, including the period-doubling bifurcation, the Neimark–Sacker bifurcation, the Neimark–Sacker-2 bifurcation, the period-X bifurcation, and the Neimark–Sacker-X bifurcation, in the dynamic walking of a bipedal robot with a torso coupled via hip series elastic actuators [11]. We showed recently in [30,31] that the semi-passive dynamic walking of a controlled biped robot with a torso as an upper body displays the Neimark–Sacker (torus) bifurcation. We employed in [31] the Lyapunov exponents as a significant tool to investigate order and chaos.

Active control of biped robots can enlarge the basin of attraction of passive limit cycles and can create new gaits [3]. There are some research works focused on the control of chaos and bifurcations in dynamic walking of biped robots [14–16]. Generally, the Poincaré map and its linearization are the two main keys employed for the synthesis of the controller [29,32–34]. Recently in [35,36], we developed a control strategy based on the OGY approach to suppress chaos exhibited in the passive dynamic walking of the compass-gait model and the torso-driven biped robot. Our control process was achieved mainly by designing an analytical expression of a controlled hybrid Poincaré map. Such hybrid map was obtained analytically by linearizing the algebro-differential equations around a desired hybrid limit cycle embedded within a chaotic attractor for some bifurcation parameter. The hybrid Poincaré map was found to mimic the impulsive hybrid nonlinear dynamics. Thus, via the controlled hybrid Poincaré map, we designed an OGY-based state-feedback control law to stabilize its period-1 fixed point.

In our previous work [13], we analyzed the displayed behavior in the impulsive hybrid nonlinear dynamics of the compass-gait model under the OGY-based state-feedback control. Under such control, the controlled dynamics is still an impulsive hybrid nonlinear system, which is found to be more complex for analysis than that uncontrolled. We showed through bifurcation diagrams that the walking dynamics of the biped robot under control displays several interesting nonlinear phenomena, namely period-doubling bifurcations, cyclic-fold (saddle-node) bifurcations, chaos, period doubling, period remerging and period bubbling. Our investigation in [13] was limited only to the steady gaits, i.e. periodic and chaotic motions. For analyzing stability of periodic gaits, we employed the perturbation technique, which is based on the linearized Poincaré map. We showed that such method is not efficient for computing fixed points and their stability near the bifurcation points. We showed also in [13] that the controlled hybrid Poincaré map can be used in order to investigate the displayed nonlinear phenomena in the impulsive hybrid nonlinear dynamics of the compass-gait model under control. In [37], we proposed a method to compute the spectrum of Lyapunov exponents in the compass-gait model under the OGY-based control method. In [14], we analyzed the walking dynamics of the compass-gait biped robot under the OGY-based state-feedback control via the controlled hybrid Poincaré map. Through this map, we studied the displayed local bifurcations and computed the branches of stable and unstable fixed points. As a result, we showed the existence of subcritical and supercritical period-doubling (flip) bifurcations, saddle-node (cyclic-fold) and saddle-saddle bifurcations, and a new type of local bifurcation, namely the saddle-flip bifurcation. Furthermore, in order to obtain an in-depth explanation on the abrupt birth/death of the local bifurcations and hence the sudden appearance/disappearance of steady gaits, we introduced a two-parameter bifurcation diagram. Thanks to this diagram, we showed that some new hidden attractors and hence new steady gaits are displayed for large values of the bifurcation parameter, the slope angle. Actually, the concept of hidden attractors in chaotic systems is important and potentially problematic in engineering applications and it has been widely investigated, as for example in [38–40].

In this paper, we show for the first time the exhibition of the Neimark–Sacker-1 bifurcation in the walking dynamics of the compass-gait model under the OGY-based state-feedback control. Such bifurcation is localized by means of the two-parameter bifurcation diagram. Our analysis is achieved via the controlled hybrid Poincaré map instead of the impulsive hybrid dynamics model as in [14]. In fact, in [14], we determined a generalized expression of such discrete hybrid Poincaré map under control permitting us to find period-p fixed points, which can be considered (approximately) as that of the period-p hybrid limit cycle of the controlled impulsive hybrid nonlinear dynamics of the compass-gait model. In the present study, we provide an analytical expression of the hybrid Poincaré map under control to find the period-1 fixed point and hence to localize the Neimark–Sacker bifurcation. Moreover, we derive an expression of the Jacobian matrix of the controlled hybrid Poincaré map in order to analyze stability of the period-1 fixed point. We show that such Jacobian matrix is expressed in terms of a single parameter, which is the impact instant. In addition, we provided three relations used to localize the Neimark–Sacker bifurcation and the associated period-1 fixed point. Using bifurcation diagram, we demonstrate the rise of the Neimark–Sacker bifurcation, which is found to be raised from a period-1 stable fixed point. Moreover, we employ the 2D phase-plane portraits of the discrete hybrid Poincaré map to further analyze transformation of the period-1 fixed point into a closed invariant circle via the Neimark–Sacker bifurcation and hence into a more complex shape as the bifurcation parameter (slope) varies. In addition, we use the two-parameter bifurcation diagram to show the occurrence of the Neimark–Sacker bifurcation with respect to the nominal slope parameter.

The present work is divided into five sections. Section 2 provides a brief description of the compass-gait model, its impulsive hybrid nonlinear dynamics and its OGY-based state-feedback control. In Section 3, we provide conditions for the localization of the Neimark–Sacker bifurcation using the analytical expression of the controlled hybrid Poincaré map. Section 4 is devoted for the investigation and hence demonstration of the emergence of the Neimark–Sacker bifurcation. In Section 5, we analyze the walking dynamics of the compass-gait model under the OGY-based state-feedback control using the 2D phase-plane diagrams and the time series. Some discussions and further comments are given in Section 6. Finally, the conclusion at the end sumps up all the work and some possible future directions are noted.

2. The compass-gait model and its OGY-based state-feedback control

2.1. Model description

The compass-gait biped robot model is illustrated by Fig. 1 where its significant parameters used in the numerical simulation are given in Table 1. The compass-gait model is composed of two