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Uncertain impulsive Lotka–Volterra competitive systems: Robust stability of almost periodic solutions

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ABSTRACT

Models with uncertain values of their parameters are of a practical significance in different fields of science and technology. Uncertainty in the competitive models can greatly affect their dynamical behavior and, therefore, the analysis of the effects of uncertain terms in such models is very important in both theory and application. In this paper, we extend the existing *N*-species impulsive competitive models to the uncertainty case. The existence of a unique strictly positive and robustly exponentially stable almost periodic solution is investigated for the model. The main results are obtained by using Lyapunov-type functions and a comparison principle.

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1. Introduction

The Lotka–Volterra type competitive systems are very important in the models of multi-species population dynamics. It is also important to note that Lotka–Volterra-type models with a competitive dynamic can be made by means of observations and can be directly extracted from real world time series. See, for example, [1–3], and the references therein. These kinds of systems are of a great interest not only for population dynamics or in chemical kinetics, but they establish the basis of many models studied in optimal control, biology, medicine, bio-technologies, ecology, economics, engineering, neural networks, etc. Such applications heavily depend on the dynamic behavior of systems; therefore, the qualitative analysis of these dynamic behaviors is a necessary step for practical applications of competitive systems [4,5].

However, it is generally recognized that some kinds of impulsive effects are inevitable in population interactions. The ecological system is often deeply perturbed by human exploit activities such as planting and harvesting etc. Also, instantaneous reduction of the population density of a given species is possible after its partial destruction by catching or poisoning with chemicals used at some transitory slots in fishing or agriculture. As a result the growth of species often undergoes some discrete changes of relatively short time interval at some fixed times. These perturbations cannot be considered continually in order to give an accurate description of the system. Introducing the impulsive effects, which cannot boil down to stochastic, random perturbations such as regime switching [6] or noisy phenomena [7] from its definition, into the model may describe such phenomena well, see [8,9]. We note that impulsive disturbances can result in changes of the biological parameters in Lotka–Volterra systems [10–12]. Similar effects have some other types of situations such as data aggregation [13]. But, it is particularly important to mention that the impulses can also be considered as control phenomena. In the past decades, impulsive control has been shown to be a powerful tool in the theory and applications of nonlinear dynamical systems. Indeed, impulsive control arises naturally in a wide variety of applications. Various good impulsive control approaches have been proposed in many fields including population growth and biological systems [14–17]. There are many cases where impulsive control can give better performance than other existing control strategies, such as feedback control [18], fuzzy control [19] or sampled-data control [20] and some others. Sometimes even only impulsive control can

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be used for control purpose. Studies have shown that impulsive control is attractive because it needs small control gains and acts only at discrete times; thus control cost and the amount of transmitted information can be reduced drastically [21]. For example, in the case of infections on the species an impulsive control strategy could minimize the amount of species injury which justifies the cost of using controls or to guarantee the use of the lowest species density which causes economic damage. On the other side, the phenomena associated with impulsive perturbations may also lead to uncontrolled situations, bifurcation or chaos. See, for example [22]. It is therefore clear, that the study of the effect of pulse phenomena on the behavior of Lotka–Volterra models is of a great importance to the theory and applications.

Since periodicity of biological and population dynamics models is an important factor, it has been studied by many authors. See, for example, [23–25] and the references cited therein. Also, biological models with almost periodic coefficients have been studied extensively [9,26–31]. It is well known that the assumption of almost periodicity of the system coefficients is a way of incorporating the time-dependent variability of the environment, especially, when the various components of the environment are periodic, not necessarily with commensurate periods (e.g. seasonal effects of weather, food supplies, mating habits and harvesting).

While the importance of accounting for almost periodicity of the coefficients when making environmental or/and management decisions subject to resource constraints has been increasingly recognized in recent years, the uncertainty associated with such decisions has often been ignored. Uncertainty in the competitive models may be caused by the relative and combined impacts of climate change, land-use change, and altered disturbance regimes on species' extinction risk. Each modeling component introduces its own source of uncertainty through different parameters and assumptions, which, when combined, can result in compounded uncertainty that can have major implications for management. Generally speaking, these uncertainties may lead to oscillation, divergence, or instability which may be harmful to the system, just as impulses would be, even if the uncertainties are tiny [32–34]. For some papers on the impulsive models with uncertainties we refer to [35,36]. In addition to these, the books of Martynyuk [37] and Weinmann [38] are good sources for the theory of uncertain systems of ordinary differential equations and uncertain impulsive systems, as well as their numerous applications. In the paper [39] the uncertainties in an impulsive control of Lotka–Volterra predator-prey system are considered and some sufficient conditions of the asymptotic stability are established. However, to the best of our knowledge, no findings about almost periodicity results for uncertain impulsive competitive multi-species systems have been presented in the literature.

Motivated by the above discussion we formulate an uncertain impulsive N-species Lotka–Volterra competitive model. The purpose of this paper is to provide a mathematical framework that encompasses the uncertain case, and to elucidate system's parameters controlling the almost periodic behavior. This research serves as a first step to extend the current impulsive N-species competitive models to the uncertainty case. The time periodic case can be taken care of as a sub-class of the model that we consider in this paper.

The paper is organized as follows: Section 2 introduces the impulsive uncertain model and some essential notations and definitions. In Section 3, conditions for the existence of almost periodic solution and its robust asymptotic stability are obtained. Piecewise continuous Lyapunov functions have been utilized to prove the main results. An example is given in Section 4 to demonstrate the effectiveness of the results.

2. Preliminaries

Let $\mathbb{R}^N$ be the N-dimensional Euclidean space with norm $||x|| = \sum_{i=1}^N |x_i|$, $\mathbb{R}^+ = [0, \infty)$, $S_\nu = \{x \in \mathbb{R}^N : \ ||x|| \le \nu\}$, $\nu > 0$, and let $\mathbb{B} = \{\{\tau_k\} : \ \tau_k \in \mathbb{R}, \ \tau_k < \tau_{k+1}, \ k \in \mathbb{Z}, \ \lim_{k\to\pm\infty} \tau_k = \pm\infty\}$ be the set of all sequences $\{\tau_k\}$ which are unbounded and strictly increasing with distance $\rho(\{\tau_k^{(1)}\}, \{\tau_k^{(2)}\})$, let $PC = PC[\mathbb{R}, \mathbb{R}^N] = \{\varphi : \mathbb{R} \to \mathbb{R}^N, \ \varphi$ is a piecewise continuous function with points of discontinuity of the first kind τ_k, $\{\tau_k\} \in \mathbb{B}$ at which $\varphi(\tau_k^-)$ and $\varphi(\tau_k^+)$ exist, and $\varphi(\tau_k^-) = \varphi(\tau_k)\}$.

Consider the following impulsive multi-species uncertain Lotka–Volterra system

$$\begin{cases} \dot u_i(t) = u_i(t)\left[r_i(t) - \sum_{j=1}^N \Big(a_{ij}(t) + b_{ij}(t)\Big)u_j(t)\right], & t \ne \tau_k, \\ \Delta u_i(\tau_k) = (p_i + q_i)u_i(\tau_k), & k \in \mathbb{Z}, \end{cases} \tag{2.1}$$

where $i = 1, 2, \dots, N$, $N \ge 2$ is the number of the competing species, $u_i(t)$ are the sizes of the populations at a given time t, $r_i(t)$ are the inherent per-capita growth rates at the moment t, $a_{ij}(t)$ represents the effect that species j has on the population of species i at the moment t, $b_{ij}(t)$ are the uncertain terms added to the interaction coefficients, and: a) The functions $r_i(t) \in C[\mathbb{R}, \mathbb{R}^+]$, $1 \le i \le N$, and a_{ij}, $b_{ij} \in C[\mathbb{R}, \mathbb{R}^+]$, $1 \le i, j \le N$; b) $\Delta u_i(\tau_k) = u_i(\tau_k^+) - u_i(\tau_k^-)$, and the constants $p_i, q_i \in \mathbb{R}$, $1 \le i \le N$, $\{\tau_k\} \in \mathbb{B}$, $k \in \mathbb{Z}$, where $u_i(\tau_k^-)$ and $u_i(\tau_k^+)$ are, respectively, the population sizes of the ith species before and after an impulsive perturbation at the moment τ_k, the constants p_i represent the abrupt changes of the population sizes at the impulsive moments τ_k, and q_i are the uncertain terms in the impulsive part of the model.

The functions $b_{ij}(t)$ and the sequence $\{q_i\}$, $1 \le i, j \le N$, $k \in \mathbb{Z}$, which represent the structural (bounded by certain limits) uncertainties [35,37,38] or uncertain perturbations, are characterized by

$$b_{ij} \in U_b = \left\{ b_{ij} : b_{ij}(t) = e_{b_{ij}}(t).\delta_{b_{ij}}(t), m^L_{b_{ij}}(t) \le \delta_{b_{ij}}(t) \le m^R_{b_{ij}}(t) \right\},$$

and

$$q_i \in U_q = \left\{ q_i : \ q_i = e_i.\delta_i, \quad |\delta_i| \le |m_i| \right\}, \quad k \in \mathbb{Z},$$

where $e_{b_{ij}} : \mathbb{R} \to \mathbb{R}^+$ are known smooth functions, and $e_i \in \mathbb{R}$ are known constants. The functions $\delta_{b_{ij}}(t)$ and the constants δ_i are unknown bounded functions and unknown bounded constants, respectively, by the functions $m^L_{b_{ij}}(t)$, $m^R_{b_{ij}}(t)$ and the norms of the constants $m_i \in \mathbb{R}$, respectively. Here $m^L_{b_{ij}}, m^R_{b_{ij}} : \mathbb{R} \to \mathbb{R}^+$ are given smooth functions.

Let $t_0 \in \mathbb{R}$ and $u_0 = col(u_{10}, u_{20}, \dots, u_{N0})$, $u_{i0} \in \mathbb{R}$ for $1 \le i \le N$. Denote by $u(t) = u(t; t_0, u_0)$, $u(t) = col(u_1(t), u_2(t), \dots, u_N(t))$, the solution of (2.1) with the initial condition

$$u(t_0^+; t_0, u_0) = u_0. \tag{2.2}$$

The solution $u(t) = u(t; t_0, u_0)$ of problem (2.1), (2.2) is a piecewise continuous function with points of discontinuity of the first kind at the moments τ_k, $k \in \mathbb{Z}$, at which it is continuous from the left, i.e. the following relations are valid:

$$u_i(\tau_k^-) = u_i(\tau_k),$$

$$u_i(\tau_k^+) = u_i(\tau_k) + (p_i + q_i)u_i(\tau_k), \quad k \in \mathbb{Z}, \quad 1 \le i \le N.$$

The problems of existence, uniqueness, and continuability of the solutions of impulsive differential equations have been investigated by many authors. Efficient sufficient conditions which guarantee the existence of the solutions of such systems are given in [8,12].

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