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Detecting chaos and predicting in Dow Jones Index

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ABSTRACT

A new theory for characterization of chaos is the basis for a chaos approach in Econophysics. Distinct periods of Dow Jone Index are the objects of study in the reconstruction scheme. They include the Economic Crashes of 1929 and 1987. The computational routines analyze the time series of stock market indices in the Algebraic Computational environment. The method developed distinguishes between chaos and randomness from real systems. This paper presents conclusive results about the dynamic characteristic of Dow Jones Index evolution.

1. Introduction

Nowadays it is reasonable to admit that Econophysics presents significant contributions to Financial Economics. Recent works have been giving significant contributions to literature in this interdisciplinary area of knowledge [1–3]. In the understanding of stock price and return variations, the power-law distributions lead to a characterization for the occurrence of extreme values. Practical implementations, the identification of a critical exponent, the assumptions about volatility, long-term perspectives, the prediction of financial crashes – and their possible management – play a valuable role even in trading rooms [4–7]. Another branch of study for the financial markets is the chaos approach. However, in this proposal, it is necessary to distinguish between chaotic time evolution and a random process [8]. Fortunately, there is a new method which characterizes the dynamics from a time series in the reconstruction's scheme. It provides alternatives to Lyapunov exponents and leaves signatures of chaos or randomness in diagrams [9].

One can consider the analysis of time series – from real systems, in terms of nonlinear dynamics – as the most direct connection between chaos theory and the real world [10]. The Algebraic Computation is so convenient in this line of research. In fact, the predictability for *chaotic time series* has been applying in this computation environment successfully [11–14]. This paper looks at the ordered set of Dow Jones Index from this methodology. Both dynamic characterization and forecast are the means for extracting information of the stock market's time series [15,16]. The next section explains the theoretical basis of this work and revisits our theory for a new characterization of chaos [9]. It describes the forecast's theory, the graphical representations of the dynamics and the new quantifier of chaos employed. Routines, computational procedures and their results for the stock market are the content of the third part of this article. Finally, the fourth section presents the discussions about the results obtained.

2. Forecasting and dynamic characterization revisited

A convenient methodology for predicting an observable has support on the theorems proved by Takens [17]. If a Euclidean space has dimension $d_E = 2m+1$, then a compact manifold of dimension m can be embedded in this space [18]. For this purpose, it suffices to collect elements of a time series $\{X(0), X(\Delta\theta), \ldots, X((N-1)\Delta\theta)\}$ with a *delay time T*. They will be components of a state vector $|\psi\rangle$ into a d_E-dimensional Euclidean space which reconstructs the original phase space – that is unknown, by definition [10,19,20]. Then the predictor $\mathcal{P}_\tau(|\psi(t)\rangle)$ approaches well the observable $X(t+\tau\Delta\theta)$ from a vector corresponding to the past time t. In this modeling, the time interval between two observables is $\Delta\theta$. The integer parameter τ specifies the *prediction time* $\tau\Delta\theta$.

$$|\psi(t)\rangle \doteq \begin{bmatrix} X(\,d_E\,T\,\Delta\theta\,) \\ \vdots \\ X(\,2T\,\Delta\theta\,) \\ X(\,T\,\Delta\theta\,) \end{bmatrix} \tag{1}$$

$$X(t+\tau\Delta\theta) \cong \mathcal{P}_\tau(|\psi(t)\rangle) \tag{2}$$

The global approach technique employs the least squares method for determining the parameters of the predictor. There is a

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mathematical convenience in the minimization process if the predictive function is linear in the adjustment parameters [12]. In this work, the reconstructed space has four dimensions with the linear functional form

$$\begin{aligned}\mathcal{P}_\tau(X_1, X_2, X_3, X_4) = {} & a_1X_1 + a_2X_2 + a_3X_3 + a_4X_4 \\ & + a_5 \ln\left(1 + \frac{1}{10}\cos(X_1)\right) \\ & + a_6 \ln\left(1 + \frac{1}{10}\cos(X_2)\right) \\ & + a_7 \ln\left(1 + \frac{1}{10}\cos(X_3)\right) \\ & + a_8 \ln\left(1 + \frac{1}{10}\cos(X_4)\right) \\ & + a_9 \ln\left(1 + \frac{1}{10}\sin(X_1)\right) \\ & + a_{10} \ln\left(1 + \frac{1}{10}\sin(X_2)\right) \\ & + a_{11} \ln\left(1 + \frac{1}{10}\sin(X_3)\right) \\ & + a_{12} \ln\left(1 + \frac{1}{10}\sin(X_4)\right) \qquad (3)\end{aligned}$$

for all calculations.

The deviation in the global fitting, given by

$$\sigma_\tau = \left\{ \frac{\sum_{r=1}^{N}\left(X_{1r} - \sum_{i=1}^{n} a_i\omega_i(|X_{r-\tau}\rangle)\right)^2}{N-1} \right\}^{\frac{1}{2}}, \qquad (4)$$

is the basis for the construction of our complex quantifier of chaos $\mathcal{Z}_{dyn}$. It is the square root of the sample variance. Above, only linear predictors in the adjustment parameters take part in the method. So we have $\mathcal{P}_\tau(|\psi\rangle) = \sum_{i=1}^{n} a_i\omega_i(|X\rangle)$ [12,14].

$$\mathcal{Z}_{dyn} = A_1 + i\,\lambda_{dyn} \qquad (5)$$

Here, $i = \sqrt{-1}$ is the imaginary unit and $\{A_1, \lambda_{dyn}\}$ are the statistical magnitudes of interest. In the first, we put $\tau = 1$ in the *accuracy for a prediction time* A_τ [9].

$$A_\tau = \frac{\sum_{r=1}^{N-1}|X_r|}{3(N-1)\sigma_\tau} \qquad (6)$$

Another useful quantity in this scheme is the *relative deviation for a prediction time* D_τ, defined by the rate

$$D_\tau = \frac{\sigma_\tau}{\sigma_1}. \qquad (7)$$

The mean λ_{dyn} provides a quantifying for the predictability of a time series. In chaotic systems, this magnitude must be positive because the accuracy in forecasting decays with the increasing of the prediction time $\Delta\theta$. Its asymptotic behavior is $\lambda_{dyn} \to 0$ if the time series is periodic or random [9].

$$\lambda_{dyn} = \sum_{\tau=1}^{M-1} \frac{A_\tau - A_{\tau+1}}{M-1} \qquad (8)$$

A powerful visual aid for the dynamic characteristic of a time series is the *Diagram Accuracy–Deviation*. It plots the logarithms $\{\ln A_\tau, \ln D_\tau\}$ as functions of the parameter τ [9]. Fig. 1 shows the corresponding diagrams for a chaotic voltage (see Fig. 1(a)) and random numbers (see Fig. 1(b)).

3. Applying the method in Dow Jones Index

Three computational routines are responsible for the treatment of Dow Jones Index in the chaos approach perspective. The dynamic characterization employs the program DynCharTS [9,21],

Table 1
Quantifying of chaos in Dow Jones Index. In this table, **NA** abbreviates *not applicable*.

Time Series	Date	Designation	$\mathcal{Z}_{dyn}$	
			A_1	λ_{dyn}
I	20 September 1898	Early Stock Market	30.2	2.00
II	2 March 1929	Before Great Depression	31.1	1.53
III	29 October 1929	Black Tuesday	NA	NA
IV	21 December1934	During Great Depression	16.8	0.95
V	30 December 1950	None	48.3	3.17
VI	30 December 1960	None	58.4	4.20
VII	31 December 1970	None	53.1	3.58
VIII	31 December 1980	None	35.2	2.29
IX	19 October 1987	Black Monday	NA	NA
X	29 December 2000	None	31.5	1.99
XI	15 September 2008	Financial Crisis	32.3	1.78
XII	1 August 2017	None	46.3	3.08
XIII	NA	Chaotic Circuit	88.1	7.80

the LinGfiTS routine provides the global map for the forecasting tasks and the ConfiTS procedure calculates the deviation σ_τ (4) beyond to analyze the residuals in the fit [12,14]. They run in a Maple environment from the commands reproduced below.

```
[> Z := DynCharTS(Data = DowJones, Dim = 4, Func = p,
        Final = 21179, Level = 20, PT = 12);
[> M := LinGfiTS(V, 21179, Func = p, Level = 20);
[> S := ConfiTS(M, V, 21179, Level = 20, Analysis = 1);
```

The arguments above specify the time series (Data), the dimension of phase space reconstruction (Dim), the functional form for the predictor (Func), the last index for analysis (Final), the interval of time series studied (Level), the maximum parameter τ for Dynamic Characterization (PT), the list of vectors employed in Global Fitting (V), the global map (M) and the analysis' option (Analysis=1).

Distinct periods of the Dow Jones Index history are in our scope of analysis. The analysis covers thirteen set of data. The chaotic voltage take part in this study for comparison with the stock market (see Table 1). Each one of these time series has 648 observables. The predictor (3) – in a four-dimensional Euclidean space – is the basis for all analysis and forecasting.

3.1. Results

Fig. 2 shows the typical diagram for almost all time series studied and the dynamic characteristic of the Economic Crash of 1929. Table 1 presents the statistical magnitudes of the chaos quantifier $\mathcal{Z}_{dyn}$ while Table 2 shows the results related to forecasts for each period analyzed. They cover twelve values for the parameter τ. So the prediction times vary from one to twelve days in the stock market activity. Other two graphical representations of results from our method are in Fig. 3. It presents both the quality of a global mapping – with the parameter $\tau = 1$ – and the accuracy in the prediction for the interval finished on 31 December 1970 (period **TS** VII). The content of Fig. 4 is the residuals' distribution in the global mapping of the series **TS** II and **TS** VII.

3.2. On the robustness of global approach

A relevant topic for both forecasting and dynamic characterization is the robustness of the method. Determining of predictor employs the least squares minimization in global fitting. It is quite practical for our programs [12,14,21]. Even so, this mathematical convenience must be subject to *diagnostics* because deviations from assumed assumptions can be present [22]. Strictly speaking, the predictors should be free of *large residuals* and produce the