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Elastodynamic Doppler effects by a moving stress-free edge

Kazumi Watanabe *

Department of Mechanical Engineering, Graduate School of Science and Engineering, Yamagata University, Yonezawa, Yamagata 992-8510, Japan

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ABSTRACT

Wave reflection by a uniformly moving stress-free edge is discussed for all three types of incident plane SH, P and SV-waves, and the Rayleigh surface wave on the moving edge is also discussed. Focusing on the Doppler effect and ray angles for the reflected wave, all frequency shifts and reflection angles are expressed in the closed form of simple algebraic equations. It is found that the Rayleigh surface wave on the moving stress-free edge has its complex velocity. This complex velocity means that the Rayleigh wave decays when the edge is expanding, but grows exponentially when the edge is shrinking. This is a completely new result.

1. Introduction

The Doppler effect is one of well-known wave phenomena. Frequency change is perceived when a source, receiver or reflector is moving. The Doppler effect by a moving reflector, so-called "scattering Doppler effects" or "moving mirror problem," is a very important sensing principle for detecting a moving object. Very extensive theoretical and experimental works have been carried out for electromagnetic and ultrasonic waves, and many kinds of sensing instruments are also developed, so far. Among the works, theoretical ones by Censor [1–4] and by De Zutter [5–9] and the recent experimental work by Wunenburger et al. [10] should be cited. They discussed 2 and 3D problems for a non-uniformly moving reflector/interface.

Elastodynamic problem for a moving source in and on an elastic solid is important, too. Dynamic response of bridges and rail roads to running vehicles [11] and the ground response to landing and take-off of airplanes are typical examples. However, the Doppler effect for the elastic wave did not attract much attention because engineering interests were paid on the magnitude of the response, less on its frequency components. Although the work on the Doppler effect for elastic waves is very scarce, some academic interests still remain due to the coupling nature of elastic waves. In addition, owing to the recent development of smart and intelligent technology, more elaborate techniques for the remote sensor might be needed in the future. The scattering Doppler effect is one of hopeful candidates for monitoring dynamic behavior of structures. These are the motivation of the present work.

The author [12] considered and summarized the Doppler effects by a uniformly moving load on a plate and a source in elastic solids. The measurability of a beam vibration by means of the Doppler frequency shift in 1D longitudinal wave [13] was also discussed. The present paper considers the scattering Doppler effect by a uniformly moving stress-free edge. As shown in Fig. 1, when an elastic solid undergoes some dynamic loads and deforms dynamically, its edge moves with some velocity. At this instant, if we input a wave toward the moving edge, a receiver will get its reflected wave with frequency shift. Analyzing the frequency shift with some data processing units, one will be able to detect the motion of the edge,

* Tel.: +81 (0)238 26 3210; fax: +81 (0)238 26 3205.
E-mail address: kazy@yz.yamagata-u.ac.jp.

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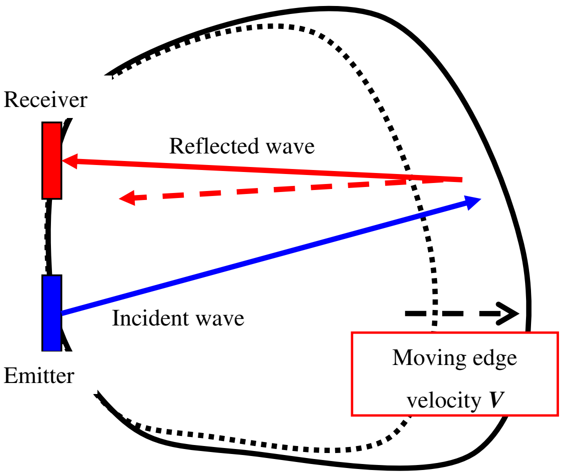

Fig. 1.1. A concept for detecting a moving boundary.

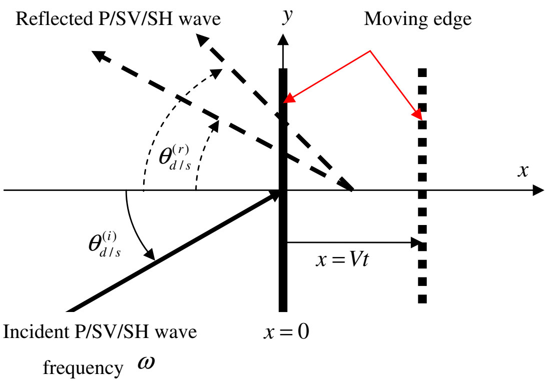

Fig. 1.2. Wave reflection at a moving edge.

i.e. a deformation sensor. This is a hopeful application of the present work. However, we have to recognize that the deformation induces some changes in elastic properties because the exact treatment for the present problem is essentially nonlinear, such as acoustic waves [3,10]. If we consider one instant during the dynamic deformation and emit an incident wave, the nonlinearity will be let aside because the velocity of the elastic wave is much larger than that of the deformation. Otherwise, our linear treatment will give a first order approximation for this problem, at least. Then, the present paper employs the linear theory of elastodynamics for discussing the scattering Doppler effect.

The problem discussed here is schematically shown in Fig. 1.2. We assume that a stress-free edge of an elastic half space is moving with uniform velocity V. An oblique incident wave is emitted from the left side ($x < 0$), then the waves are reflected by the moving edge. Throughout the paper, incident and reflection angles are classified by superscript (i) and (r), and the wave type, such as dilatational and shear waves, by the subscripts d and s. The notation ω is used for the frequency of all incident waves, and ω_d and ω_s are for those of reflected dilatational and shear waves, respectively. We also assume that the moving edge is always parallel to the y-axis and is $x = 0$ at $t = 0$ in a (x, y) plane. Further, the present paper assumes that all Mach numbers are less than unit one. This assumption will be appropriate for engineering applications.

Similar works for electromagnetic wave have already been carried out by Yeh [14], Daly and Gruenburg [15] and Huang [16]. But, these are concerned only with the scalar wave and no coupling effect between dilatational and shear waves is discussed. Thus, the present discussion on P, SV and Rayleigh waves is a new one for the Doppler effect. Then the author proposes the name "Elastodynamic Doppler effects" for the present work. The present paper discusses the reflection of SH-wave wave reflection in Section 2, that of P-wave in Section 3 and of SV-wave in Section 4. As a natural extension of interest, the Rayleigh surface wave which is running on the moving edge is discussed in Section 5. The Section 6 is conclusion.

2. Incident SH-wave

SH-wave is a simple scalar wave and its governing equation is just the same as that of electromagnetic and acoustic waves. Mathematics and results for the Doppler effect by a moving edge are also the same as those of the moving mirror