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Imperfect chimeras in a ring of four-dimensional simplified Lorenz systems

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ABSTRACT

Analyzing the dynamics of networks of coupled oscillators possessing chimera states has attracted considerable attention in recent years. Here we study a ring of coupled 4D simplified Lorenz systems whose prototype was first studied by Li and Sprott [1]. An interesting feature of this system is the coexistence of a limit cycle and two symmetric strange attractors for some specific range of parameters. Investigating a network of this type under long range interactions, we find a multitude of very interesting chimera multi-chimera states, whose appearance we relate to initial conditions chosen within the basins of attraction of the limit cycle and strange attractor of the individual 4D system. Thus, for sufficiently low coupling, selecting more and more initial conditions from the strange attractor basins leads to one or more "rebel" particles departing from the synchronous group and producing fascinating "imperfect" chimera states. For larger coupling these "rebels" multiply and the system develops what one might call "perfect" chimera states with multiple "heads".

1. Introduction

Chimera state is a fascinating topic in nonlinear dynamics which has attracted great interest in recent years [2–5]. This state characterizes situations where synchronous and asynchronous clusters coexist in networks of identical coupled oscillators [6–8]. There is evidence that confirms the relevance of chimera states in real - life phenomena, such as the uni-hemispheric sleep observed in birds and dolphins, where one hemisphere of the brain is synchronous and the other is asynchronous [9], while they have also been associated with some disorders in the brain such as epileptic seizures [10].

Chimera states have been studied in a great variety of dynamical systems. Abrams and Strogatz [11] first explained this phenomenon in a ring of coupled oscillators and gave it the name "Chimera", after the famous mythological creature with the heads of lion, goat and snake. Following its discovery, many researchers started investigating this phenomenon in rings of coupled oscillators or multi - population networks.

More specifically, Dudkowski et al. [12] studied an interplay between coherent and incoherent dynamics in a ring of van der Pol–Duffing oscillators, while Schmidt et al. [13] reported chimera patterns of various shapes in a two - population neuronal network. Subsequently, Majhi et al. [14] considered a two layer network of Hindmarsh–Rose neurons without any direct interactions in one layer, a two-population network of coupled pendulum-like elements was studied by Bountis et al. [15] and networks of Hindmarsh–Rose neuron models were analyzed in [16]. Chimera states have also been observed in small size networks. Maistrenko et al. [17] demonstrated the occurrence of chimera states in a small network consisting of three globally coupled pendulum-like oscillators. Experimental studies have also been performed on chimera states, as described in Martens et al. [18] on a mechanical model of coupled metronomes, as well as on coupled electronic oscillators [19], coupled map lattices [20] and chemical oscillators [21].

Depending on the various spatiotemporal patterns observed in these studies, different chimera types have been introduced. Among those, we mention the so – called breathing chimera [22], traveling chimera [23] and spiral wave chimera [24]. A different type of interesting and rare phenomenon called "imperfect chimera" has also been reported recently in a number of works [25–27]. For example, Kapitaniak et al. [25] discovered imperfect chimeras by considering coupled pendula in which a certain number of oscillators escape from the synchronized cluster, while the averaged frequency of the escaped oscillators is different from that of the synchronous ones. Also, Premalatha et al. investigated in [27] different types of imperfect chimera and imperfect synchronization in populations of non-locally coupled Stuart-Landau oscillators.

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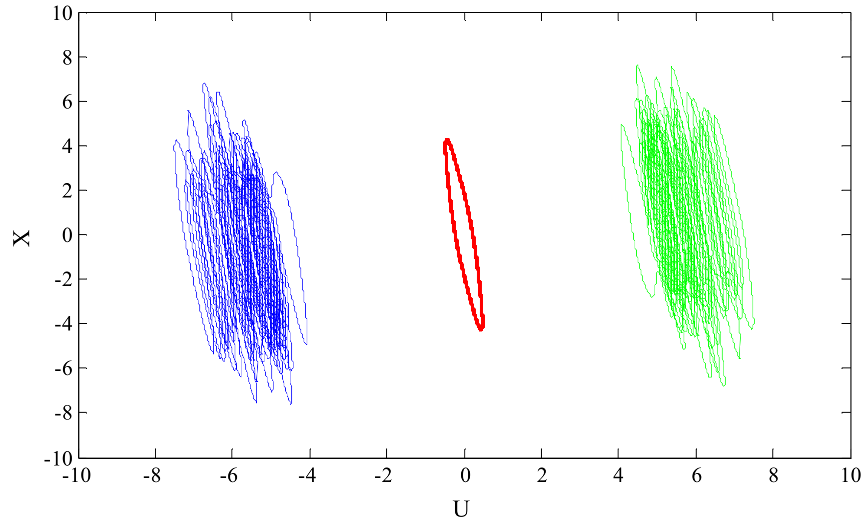

Fig. 1. An attracting limit cycle (obtained for initial conditions (−1, 0, −1, 0)) coexists with a symmetric pair of strange attractors at $a=7$, $b=0.1$ (obtained for initial conditions (±14.1, 0, 20,0).

In this paper, we study a network of coupled 4 – dimensional (4D) simplified Lorenz systems, whose prototype was originally introduced by Li and Sprott in [1]. This model was shown to possess many interesting properties, including the coexistence of stable limit cycles and strange attractors. In the present paper, we have studied a ring network composed of Li–Sprott oscillators and analyzed its spatiotemporal patterns and synchronization properties. We have thus discovered a variety of chimera states and multi-chimera states in this network's dynamics.

Our main result is that, starting from small coupling and selecting most initial conditions within the basin of attraction of a central limit cycle of the single 4D system, imperfect chimeras can be obtained by selecting one or more initial conditions from the basin of attraction of the two nearby strange attractors. Then, increasing the coupling constant, one witnesses the departure of more and more "rebel" particles and the formation of perfect chimeras. We thus conclude that in this system the asynchronous and synchronous part of chimera state can be respectively related to the presence of strange attractors and an attracting limit cycle in each one the individual elements of the network.

The paper is organized as follows: In Section 2, the dynamics of the system and the coupled network is described. Section 3 describes our main numerical results, presents examples of different chimera patterns arising in the network, and discusses how perfect chimeras emerge out of imperfect ones. Due to the importance of the central limit cycle, we present in Section 4 an analytical approximation of this solution, whose frequency provides an order of magnitude estimate of the frequency of the synchronized motion, for small enough coupling, and the parameter values chosen in our study. Finally, our conclusions and future outlook are presented in Section 5.

2. System dynamics

Let us consider a network of non-locally coupled 4D simplified Lorenz systems, each element of which has the form of a model originally proposed by Li and Sprott in [1]. Thus, every oscillator of the network is described by the equations:

$$\begin{aligned}\dot{x} &= y - x\\ \dot{y} &= -xz + u\\ \dot{z} &= xy - a\\ \dot{u} &= -by\end{aligned} \tag{1}$$

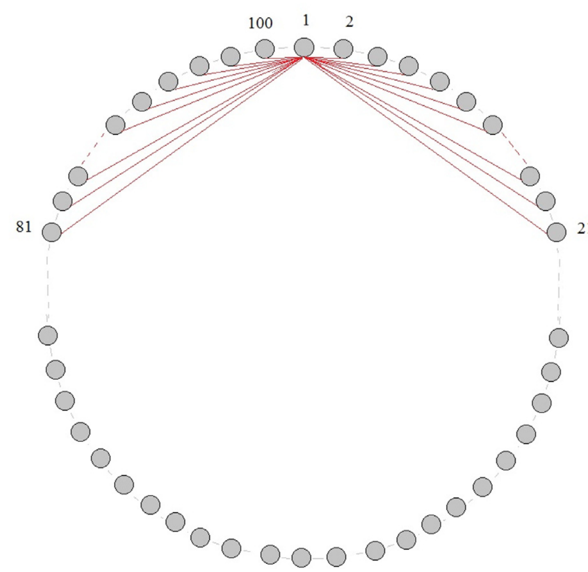

Fig. 2. Schematic of coupled oscillators on the ring. Each oscillator interacts with $P=20$ of its left and right neighbors.

In different ranges of parameter space, the system exhibits the coexistence of an attracting torus with a symmetric pair of strange attractors, a symmetric pair of limit cycles and the coexistence of a symmetric pair of strange attractors and a limit cycle. Here, we choose the latter case, and focus on the specific parameter values $a=7$ and $b=0.1$. Fig. 1 shows the three coexisting attractors for the initial conditions given in the caption.

To construct our network, we have coupled $N=100$ of above oscillators on a one-dimensional ring. The coupling is non-local, and we use nearest neighbor connections, in a way that each network element is coupled to its $2P$ nearest neighbors, as shown in Fig. 2. The equations of the coupled oscillators are given as follows:

$$\dot{x}_i = y_i - x_i + \frac{d}{2P}\sum_{j=(i-P)\mathrm{mod}N}^{(i+P)\mathrm{mod}N}\left[x_j - x_i\right],\quad 1 \le\ k\ \mathrm{mod}N \le N$$

$$\dot{y}_i = -x_i z_i + u_i$$