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A new oscillator with infinite coexisting asymmetric attractors

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ABSTRACT

In this paper, we introduce a new two dimensional nonlinear oscillator with infinite number of coexisting asymmetric limit cycles and stable equilibria. Some of these attractors are self-excited, while some others are hidden attractors. Adding a forcing term to this new system, we introduce a new chaotic system with infinite number of coexisting asymmetric strange attractor, torus and limit cycles, depending on the parameters.

1. Introduction

In this decade, attractors in dynamical systems were categorized into two groups: self-excited attractors and hidden attractors [1–3]. When the basin of attraction for an attractor involves unstable equilibrium point, we call that attractor self-excited. Otherwise, the attractor is hidden [4–6]. In many real-world dynamical systems, hidden attractors have been observed [7–9]. Design [10], localization [11–13], realization [14–16], control [17–19] and synchronization [20] of hidden attractors have attracted lots of interest in literature. The topic of hidden attractors is much related to the topic of multistability [21–25], which is a very important phenomenon in dynamical systems [26,27]. Although sometimes multistability is undesirable, it allows flexibility in the system's performance without changing parameters, and that can be used with the right control strategies to induce a switching between different coexisting states [28].

Designing new dynamical systems with unusual features has been a hot topic recently. As examples one can point out dynamical systems with no equilibrium [29–32], with stable equilibria [10,33,34], with line of equilibria [35,36], with curves of equilibria [37–39], with surfaces of equilibria [40,41], with plane of equilibria [42,43], and with non-hyperbolic equilibria [44]. Some other examples are dynamical systems with multi-scroll attractors [45,46], with multistability [47–49] and coexisting attractors [50–52] and with extreme multistability [43,53–56]. Many of the abovementioned systems have hidden attractors. One important category of chaotic systems includes periodically-forced nonlinear oscillators [57], of which the Van der Pol system in one of the oldest [58–61].

In this paper, we introduce a modified oscillator with infinite number of coexisting limit cycles. Some of these limit cycles are hidden attractors while some others are self-excited. Changing this new system to its forced version and choosing proper set of parameters, we introduce a new chaotic oscillator. In this oscillator, many of the previous limit cycles vanish while some new limit cycles, torus and one strange attractor are born (depending on the parameters). In the next section, the new oscillator is introduced and investigated. In Section 3, the forced version of this oscillator is introduced and its dynamical properties are investigated. Also with a circuit implementation, its feasibility for engineering application is shown. Finally, discussion and conclusion are given in Section 4.

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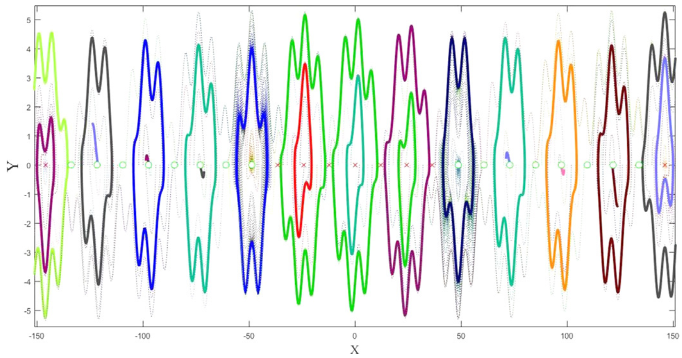

Fig. 1. Trajectories in System (1) for $K = \frac{1}{\sqrt{15}}$, for 600 initial conditions located on the x-axis (from $x = -150$ to $x = +150$ with steps equal to 0.5). The stable equilibrium points are shown by green circles, while unstable equilibrium points are shown by red crosses. Each trajectory is plotted for 1000 seconds. The first half of each trajectory is plotted with thin dots, showing the transient parts of trajectories. The second half of each trajectory is plotted with thick lines, showing the steady state of the system for the corresponding initial conditions. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

2. The new oscillator

Consider System (1),

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -sin(Kx) + \ y\cos(x) \end{aligned} \tag{1}$$

This system is a modification of System (2) [62]. We have obtained this new system by trial and error.

$$\begin{aligned} \dot{x} &= y \\ \dot{y} &= -x + \ y\cos(x) \end{aligned} \tag{2}$$

Some can simply investigate that System (1) has an infinite number of equilibrium points located at $(\frac{n\pi}{K}, 0)$ where n is any integer number.

Considering $K = \frac{1}{\sqrt{15}}$ (for which we have done most of the simulations), we continue our calculations. The Jacobian matrix for this system in its equilibria is:

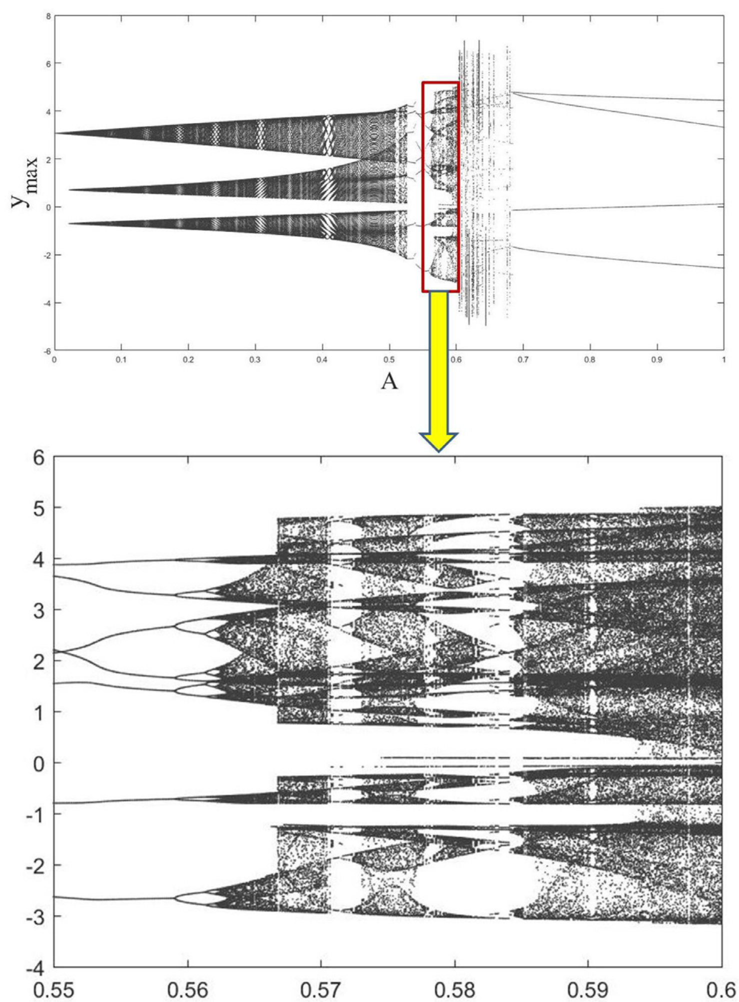

Fig. 2. Bifurcation diagram (maximum values of "y") versus forcing amplitude A in System (5) with $\omega = 0.7$. The initial conditions for every value of A were (1, 0). A zoomed version of the area between A = 0.55 and A = 0.6 can be seen in the lower part of the figure, which clearly shows the period doubling route to chaos.