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Chaotic dynamics and stability of functionally graded material doubly curved shallow shells

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ABSTRACT

In this article, the nonlinear chaotic and periodic dynamic responses of doubly curved functionally graded shallow shells subjected to harmonic external excitation are numerically investigated. Material characteristics of the shell are defined according to a simple power law distribution through the thickness. Based on the first-order shear deformation shell theory and using the Donnell nonlinear kinematic relations the set of the governing equations are derived. The Galerkin method together with trigonometric mode shape functions is applied to solve the equations of motion. Also, the nonlinearly coupled time integration of the governing equation of plate is solved employing fourth-order Runge–Kutta method**.** The effects of amplitude and frequency of external force on the nonlinear dynamic response of shells are investigated. The bifurcation diagram and largest Lyapunov exponent are employed to detect the amplitude and frequency of external force critical parameter of periodic and chaotic response of shallow shells under periodic force. Having known the critical values, phase portrait, Poincare maps, time history and power spectrum are presented to observe the periodic and chaotic behavior of the system.

1. Introduction

Functionally graded materials (FGM) are non-homogenous composites made up of ceramic and metal in which mechanical properties vary continuously from one surface to the other. The properties can be tailored via a volume fraction of the constituents. The increasing applications of FGM materials in engineering structures make them more important to study. On the other hand, having the knowledge about the dynamic response of doubly curved panel structures including the periodic, quasi-periodic and chaotic behaviors that may have practically occurred is essential in their design. The scientific concept of the chaotic phenomenon refers to those of the physical and mathematical systems that their time history has very intense and sensitive dependence on initial conditions. However, due to conformance with specific differential equations is regulated. Due to the extreme sensitivity of chaotic phenomena to initial conditions, there is a very minimal error in inputs and initial conditions, which will cause the rapid growth of error by passing the time and makes it impossible to predict.

Chaotic vibrations occur in the presence of nonlinear factors in the system. Samples of these factors include nonlinear elastic elements, nonlinear springs, nonlinear damping, looseness or vasodilatation in the system forces the fluid, nonlinear boundary conditions, carioles and center-oriented, such as the presence of geometric nonlinearity associated with the relationship between stress and strain relationships, large deformations and membrane. According to the non-linear behavior of mechanical systems for these factors, the feasibility of chaotic vibrations and calculation of parameters forming a vibrating system where chaotic behavior occurs is very important and requires researchers' attention. Extensive researches have been undertaken on the nonlinear dynamic response of structures under various boundary and loading conditions. Chaotic, quasi-periodic and periodic dynamic response analysis was investigated on various structures such as beams, plates and shells.

The chaotic response of composite plate subjected to harmonic external force was investigated by Moorthy et al. [1]. Sun and Zhang [2] carried out the chaotic dynamic buckling analysis of viscoelastic plate using chaotic and fractal theory. They considered the effects of geometry nonlinearity and viscoelastic parameter in the nonlinear dynamic re response of viscoelastic plates. The possibility of the chaotic motion of simply supported rectangular plate in large deflection was studied by Lia et al. [3] using the criteria of the fractal dimension and the maximum Lyapunov exponent. Yeh et al. [4–7] investigated the chaotic and bifurcation conditions of simply supported rectangular and circular plates of thermo-mechanical coupling by employing the criteria of the frac-

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tal dimension and the maximum Lyapunov exponent. The nonlinear vibration and chaotic behavior of an anti-symmetric cross-ply laminated composite rectangular thin plate under parametric excitation were studied by Ye et al. [8]. Ribeiro and Duarte [9] investigated the periodic and chaotic vibration of composite laminated plates under transverse harmonic forces. Touze et al. [10] used bifurcation diagrams of Poincare' maps, Lyapunov exponents and Fourier spectra analysis to investigate the transition from periodic to chaotic vibrations in free-edge, perfect and imperfect circular plates. Zhang et al. [11] presented the extended Melnikov method to detect the chaotic condition of the buckled plate.

The nonlinear dynamics response and multi-pulse chaotic behaviors of a simply supported symmetric cross-ply composite laminated rectangular thin plate subjected to transverse excitation was studied by Xiangying et al. [12]. Guo and Li [13] utilized the nonlinear ODE and the Melnikov's method to predict the chaotic conditions of a composite laminated plate under subjected to incompressible subsonic flow and transverse harmonic excitation with geometric nonlinearity. The nonlinear dynamic response and chaotic vibration of an aero-elastic piezoelectric functionally graded piezoelectric rectangular plate subjected to parametric and primary excitations were investigated by Rezaee and Jahangiri [14]. Fengxian and Chen [15] presented the Melnikov method to study of bifurcation and chaotic conditions of the nonlinear viscoelastic plates under subsonic flow and external loads.

The bifurcation diagram, phase plane, and Poincare map were employed by Dai et al. [16] to study the chaotic condition of a cantilever plate in supersonic flow. Li and Yang [17] investigated the instability and chaotic vibration of a cantilever plate with motion constraints in subsonic flow. Bifurcation and chaotic condition of thin circular functionally graded plates subjected to one-term and two-term transversal excitations in thermal environment were conducted by Yuda and Zhiqiang [18]. Nonlinear vibration, bifurcation and chaotic analysis of a simply supported functionally graded materials (FGMs) rectangular plate under transversal and in-plane excitations were studied by Zhang et al. [19]. Ribeiro [20] conducted the geometrically nonlinear vibration analysis to explore the chaotic behavior of linear elastic and isotropic plates under the combined effect of thermal fields and mechanical excitations. Fengxian and Fangqi [21] employed Melnikov method to investigate the Bifurcations and chaos of the nonlinear viscoelastic plates under the subsonic flow and external loads. The experimental study of the chaotic behavior of two-degree-of-freedom flexibly-mounted rigid plate placed in water was carried out by Modarres et al. [22]. Honghua et al. [23] studied the periodic, quasi-periodic and chaotic response of cantilever plate in supersonic flow using bifurcation diagrams. Nagai et al. [24,25] used the Fourier spectra, the Poincare projections, the maximum Lyapunov exponents and the Lyapunov dimension, to investigate the chaotic response of a shallow cylindrical shell panel subjected to gravity and periodic excitation. The control of chaotic response of an induction motor by employing the proportional integral (PI) speed loop is investigated by Messadi and Mellit [26]. Alfi [27] investigated the chaotic response of a nonlinear system by using of an optimal Hinfinity adaptive PID controller. The spatiotemporal chaotic dynamics behavior and stability analysis of a food chain model in the presence of additional food for the predator is numerically and analytically studied by Ghorai and Poria [28].

The chaotic nonlinear dynamic response analysis of doubly curved functionally graded shallow shell subjected to harmonic external excitation is the main challenges of this work in which the regular (periodic) and irregular (chaotic) parameter's domains are detected by implementing of bifurcation diagrams and largest Lyapunov exponent. The phase portrait, Poincare maps, time history and power spectrum are employed to show the periodic and chaotic nonlinear dynamic response of the system. In other words, the objective of this paper is an analysis of chaotic and periodic dynamic response of functionally graded material doubly curved shallow shells subjected to harmonic external forces. For this purpose, based on the first-order shear deformation shell theory and the Donnell nonlinear kinematic relations, the governing equation of motion is derived. The Galerkin method on the basis of trigonometric mode shape functions is applied to the governing nonlinear partial differential equation to obtain the nonlinear ordinary differential equation of motion. The ordinary differential equations are solved by the Runge–Kutta method to calculate the dynamic response of functionally graded material doubly curved shallow shells. The influences of amplitude and frequency of external excitation on the nonlinear dynamic response of doubly curved shallow shells are investigated. The bifurcation diagrams of Poincare maps and the largest Lyapunov exponent criteria are employed to detect the chaotic and periodic parameters. Moreover, the phase portrait, Poincare maps, time history and power spectrum are utilized to present the periodic and chaotic dynamic response of the system.

2. Governing equations

Fig. 1 shows a simply-supported doubly curved FG shell whose length, width and thickness are denoted by a, b and h respectively. The shell is assumed to be made of ceramic and metal graded through the thickness. To define the different material properties of the shell, the volume fractions of ceramic f_c and metal f_m corresponding to a simple power law are applied as [18].

$$f_c(z) = \left(\frac{2z+h}{2h}\right)^{\xi} \tag{1}$$

$$f_m(z) = 1 - \left(\frac{2z+h}{2h}\right)^{\xi} \tag{2}$$

where ξ is the power law exponent ($0 \le \xi \le \infty$), so that taking the value of ξ to zero indicates the rich ceramic side. The subscripts m and c represent the metal and ceramic components, respectively. Based on the mentioned power law, the optional mechanical properties of FGM can be expressed according to following [18]

$$P(z) = P_m + P_{cm}\left(\frac{2z+h}{2h}\right)^{\xi} \tag{3}$$

$$P_{cm} = P_c - P_m \tag{4}$$

So the non-homogeneous properties like Young's modulus $E(z)$ and density $\rho(z)$ are presented by the Eq. (3) and the Poisson's ratio ν is assumed to be constant through the plate thickness. The deformations are defined with the principal coordinate system (x, y, z) and the middle surface displacements are u, v, w in same directions, respectively. According to the first-order shear deformation shell theory, the normal and shear strains at distance z from the shell middle surface are given as [29,30]:

$$\varepsilon_x = \bar{\varepsilon}_x + z\kappa_x \tag{5a}$$

$$\varepsilon_y = \bar{\varepsilon}_y + z\kappa_y \tag{5b}$$

$$\varepsilon_{xy} = \bar{\varepsilon}_{xy} + z\kappa_{xy} \tag{5c}$$

where $\bar{\varepsilon}_x$, $\bar{\varepsilon}_y$ and $\bar{\varepsilon}_{xy}$ are the normal and shear strains of shell middle surface, respectively and κ_x, κ_y and κ_{xy} are the curvatures. The