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Synchronization analysis of fractional order drive-response networks with in-commensurate orders

Li-xin Yang[a,b,*], Jun Jiang[b]

[a] School of Arts and science, Shaanxi University of Science and Technology, Xi'an 710021, China
[b] State Key Laboratory for Strength and Vibration, Xi'an Jiaotong University, Xi'an 710049, China

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ABSTRACT

This paper is devoted to synchronization analysis of the fractional order drive-response complex network. Firstly, a fractional order drive-response networks model with in-commensurate orders is proposed. Moreover, on the basis of the stability theory of linear fractional-order differential equations and open-loop strategy, we derive a sufficient condition for the stability of the modified projective synchronization behavior in such drive-response complex network. Furthermore, we verify our theoretical results by numerical simulations of drive-response complex network with in-commensurate orders.

1. Introduction

Complex network can describe a wide range of systems from nature to society. Examples cover as diverse as the Internet, scientific collaboration networks, neural networks [1–3]. Understanding the effect of networks on the dynamical processes taking place on them is a central issue [4–5].

Synchronization is one of the most prevalent collective dynamics in complex networked systems. Synchronization in complex networks has attracted much attention recently [3–6]. Until now, several types of synchronization have been investigated, such as phase synchronization and complete synchronization [7,8], projective synchronization [9] and modified projective synchronization [10–12]. Among them, modified projective synchronization increase the difficulties of interception of outputting the chaotic secure communication more secure. Moreover, earlier works have found that complex network with identical order nodes can achieve various synchronization by effective strategies. However, many complex networks are not a guarantee for identical nodes. So it is necessary to investigate the modified projective synchronization of complex network with in-commensurate orders.

On the other hand, fractional-order differential systems have been widely investigated due to their potential applications in viscoelasticity, dielectric polarization, quantum evolution of complex systems, and many other fields. In fact, most studies to date have concerned integer order complex networks. The fractional order complex networks generalize well-studied integer order complex networks. It is well known that the behavior of dynamical networks with different nodes is much more complicated than the identical node case.

By summarizing the previous results, it is interesting to ask if that the drive system is an integer order system, and the response dynamical networks consisting of nodes with fractional order dynamics during the synchronization. To the best of our knowledge, no work in this aspect is available in the literature. In this paper, we focus on synchronization behavior of a fractional order drive-response complex network with in-commensurate orders.

This paper is organized as follows. In Section 2, some preliminaries of fractional calculus and drive-response complex networks with in-commensurate orders are briefly outlined. The main results for achieving modified projective synchronization of fractional order drive-response complex networks are given in Section 3. In Section 4, illustrative examples are shown to support the theory results. Concluding remarks are presented in Section 5.

2. Preliminaries and model description

2.1. Definitions of and stability theorems fractional system

There exist many definitions for fractional derivatives [13]. The Riemann–Liouvile definition and the Caputo definition are the two most commonly used ones. In this paper, the Caputo definition is adopted for derivative, which is introduced briefly below:

$$D_*^{\alpha} f(t) = \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^{t} (t-\tau)^{\alpha-n+1} f^{(n)}(\tau) d\tau, \tag{1}$$

* Corresponding author.
E-mail addresses: yanglixin@sust.edu.cn (L.-x. Yang), jun.jiang@xjtu.edu.cn (J. Jiang).

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for $n-1 \le \alpha < n$, where $\Gamma(\cdot)$ is the Gamma modified.

$$\Gamma(s) = \int_0^{\infty} t^{s-1} e^{-t} dt. \tag{2}$$

Consider the following fractional order system

$$D^q X = f(X) \text{or} D^q X = AX. \tag{3}$$

Where $X \in R^n$, $A \in R^{n\times n}$, $0 < q \le 1$.

Lemma 1. *(See [13]). If A is a constant matrix, then the autonomous linear fractional order system (3) is asymptotically stable if* $|\arg(\lambda_i(A))| > q\pi/2$.

Throughout this paper, "q -stable matrix A" means that all eigenvalues of matrix A satisfy condition (3).

2.2. Network model

Consider the fractional order drive-response complex dynamical networks as follows:

$$\dot{s}(t) = f(s(t)),$$
$$D_*^q x_i(t) = f(x_i(t)) + \sum_{j=1}^{n} c_{ij}\Gamma x_j(t) + u_i \qquad i = 1, 2, ..., N. \tag{4}$$

Where $0 < q \le 1$ is the fractional order, $x_i = (x_{i1}, x_{i2}, ..., x_{in})^T \in R^n$ denotes the state vector of the ith node, $f(x_i(t))$ are $n \times 1$ continuous differentiable functions, which describes the dynamics of the individual nodes, Γ is the inner coupling matrix, $C = (c_{ij})_{N\times N}$ is the outer coupling matrix representing the topological structure of the network, where c_{ij} is defined as follows: if there exists a link between $h_{ij}^k = 0$ node and jth node $(i \ne j)$, then $c_{ij} \ne 0$; otherwise, $c_{ij} = c_{ji} = 0$, and the diagonal elements are given by $c_{ii} = -\sum_{j=1, i\ne j}^{N} c_{ij}$, which means that the nodes are diffusively coupled.

Remark 1. In the literature, both the goal node and response network are described by integer order differential equations or fractional order differential equations, in this paper, we show that the drive system is integer order equation, but the response network is a fractional order dynamical network, in which the state variables of each node evolve with time according to a set of fractional order differential equations. That is to say, our model enlarges the drive-response complex network model.

In order to get the error dynamical complex network, the error term is defined as following:

$$e_i(t) = x_i(t) - \Lambda(t)s(t), \qquad i = 1, 2, ..., N \tag{5}$$

Where $\Lambda(t) = diag(\lambda(t), ..., \lambda(t))^T$ denotes scaling matrix, $s(t)$ means the goal orbit. For simplicity, we define a new matrix $P(t) = \Lambda(t)s(t)$.

According to the above definition, the error dynamical system is controlled by the following equation:

$$D_*^q e_i(t) = f(x_i(t)) + \sum_{j=1}^{N} c_{ij} x_j(t) - D_*^q P(t) + u_i(t)$$
$$i = 1, 2, ..., N. \tag{6}$$

The goal of this study is to design suitable controller $u_i(t)$ to synchronize the network onto a given orbit.

The vector modified $f(x_i(t))$ can be linearized as follows in the neighborhood of the goal value via Taylor expansions:

$$f(x_i(t)) = f(P(t)) + \frac{\partial f}{\partial P}(x_i - P(t)) + \cdots, \qquad i = 1, 2, ..., N. \tag{7}$$

Substituting in Eq. (7), one can obtain

$$D_*^q e_i(t) = f(P(t)) + \sum_{j=1}^{N} c_{ij} e_j(t) - D_*^q P(t) + He_i(t) + u_i(t) \tag{8}$$

where $H = \frac{\partial f}{\partial P}$ is Jacobian matrix of with respect to $P(t)$.

Our aim is to design appropriate controllers such that the drive-response network (4) with different orders can achieve the modified projective synchronization.

3. Synchronization analysis

In this section, we investigate the modified projective synchronization of fractional order drive-response network defined by Eq. (4) and design the controller via open-loop control strategy.

Before beginning our main results, some lemmas (see Appendix A for details) are needed to derive the main results.

Theorem 1. *If the coupling matrix C of the drive-response network (4) is symmetric and diffusive, the matrix* $H \otimes I_N + C \otimes I_n$ *is q-stable if and only if matrix H is q -stable.*

Proof: we prove sufficiency first.

Suppose matrix H is q -stable and ε_1, ..., ε_n are the eigenvalue of H, then $|\arg(\varepsilon_i)| > \frac{q\pi}{2}$ for $i = 1, ..., N$. Let λ_1, ..., λ_N denote the eigenvalues of matrix C. Since this matrix is symmetric and real, $\lambda_i \in R$, one can sort them out in decreasing order as $0 = \lambda_1 \ge \lambda_2 \ge ... \ge \lambda_N$. In addition, one can obtain $\lambda_i + \varepsilon_j$ for some $i = 1, 2, ..., N$, $j = 1, 2, ..., n$ are eigenvalues of matrix $H \otimes I_N + C \otimes I_n$. From Lemma 2, $|\arg(\lambda_i + \varepsilon_j)| > \frac{q\pi}{2}$. Based on Lemma 4, $H \otimes I_N + C \otimes I_n$ is q -stable.

In the following, we should proof necessity.

Assume that H is not q-stable. Thus, there exists $j \in \{1, 2, ..., n\}$ such that $|\arg \varepsilon_j| \le \frac{q\pi}{2}$. From Lemma 3, matrix C has at least one null eigenvalue, so $\rho_{i,j} = \lambda_i + \varepsilon_j = \varepsilon_j$ is an eigenvalue of $H \otimes I_N + C \otimes I_n$ such that $\arg|\rho_{i,j}| \le \frac{q\pi}{2}$, as a consequence, $H \otimes I_N + C \otimes I_n$ is not q -stable.

Hereafter, we give a useful theorem to characterizes a sufficient condition for fractional order drive-response complex network (4) to achieve modified projective synchronization.

Theorem 2. *For a certain fractional order* $q \in (0, 1]$ *and scaling matrix* $\Lambda(t)$*, the fractional order drive-response network (4) can achieve modified projective synchronization via the following controllers.*

$$u_i(t) = D_*^q P(t) - f(P(t)) - Me_i(t) \qquad i = 1, 2, ..., N. \tag{9}$$

Proof: According to errors $e_i(t) = x_i(t) - \Lambda(t)s(t) = x_i(t) - P(t)$, error dynamical system can be described by:

$$D_*^q e_i(t) = f(x_i(t)) + \sum_{j=1}^{N} c_{ij} e_j(t) - D_*^q P(t) + u_i(t)$$
$$i = 1, 2, ..., N. \tag{10}$$

We note that, the vector modified $f(x_i(t))$ is linearized as follows in the neighborhood of the goal value via Taylor expansions:

$$f(x_i(t)) = f(P(t)) + \frac{\partial f}{\partial P}(x_i - P(t)) + \cdots, \qquad i = 1, 2, ..., N. \tag{11}$$

Keeping the first-order terms in Eq. (11) and substituting in Eq. (10), we have

$$D_*^q e_i(t) = f(P(t)) + \sum_{j=1}^{N} c_{ij} e_j(t) + D_*^q P(t) + He_i(t) - f(P(t))$$
$$- Me_i(t) - D_*^q P(t)$$
$$= He_i(t) + \sum_{j=1}^{N} c_{ij} e_j(t) - Me_i(t). \tag{12}$$