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On the control of unknown continuous time chaotic systems by applying Takens embedding theory

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ABSTRACT

In this paper, a new approach to control continuous time chaotic systems with an unknown governing equation and limitation on the measurement of states, has been investigated. In many chaotic systems, disability to measure all of the states is a usual limitation, like in some economical, biological and many other engineering systems. Takens showed that a chaotic attractor has an astonishing feature in which it can embed to a mathematically similar attractor by using time series of one of the states. The new embedded attractor saves much information from the original attractor. This phenomenon has been deployed to present a new way to control continuous time chaotic systems, when only one of the states of the system is measurable and the system model is not also available.

1. Introduction

The power of system dynamics in predicting and controlling many systems urges us to study more and more in this field [1]. Chaos as a widespread phenomenon in system dynamics can endanger many systems. We have introduced a new method to control chaotic systems with limitation on measuring the states, and an unknown governing equation. Proposing a control method for these kinds of systems is a big leap toward controlling continuous time chaotic systems, because there are indeed numerous chaotic systems in which their states may not be measured practically, like many biological, economical and many other engineering-type chaotic systems [2–5]. The proposed method can also expose potentially a new vision for controlling high-dimensional chaotic systems in which usually the model of the system is not available and only a few states of the system can be available.

Chaos control in continuous time systems has had some many breakthroughs during the last decades. The delayed feedback method as one of the best control methods can be applied to many chaotic systems [6]. But these methods simply assumed the availability or at least observability of all of the states and a known governing equation on the dynamics of the system. The method presented here eradicates these assumptions. One measurable state is enough in this method to systematically design a proper control law.

The control method presented here is backed by works done by Takens [7] on chaotic attractors. Takens theorem guarantees preservation of the topological characteristics of an embedded chaotic attractor. The embedded attractor is made up by time series of measurements of a single measurable state. This is our clue to elicit some information from an unknown dynamical system.

The embedded chaotic system is not only topologically similar to the original systems but also can preserve dynamical characteristics like unstable orbits or fixed points. Our idea is to stabilize unstable fixed points (UFP) in the Poincare map of the embedded attractor and find a proper control signal for the original system with special considerations. Controlling chaotic systems with these strict conditions (unknown dynamics and limitation on measuring the states) has been previously done for discrete-time systems [8]. Working on continuous time systems unlike discrete-time systems has much more complexity in Takens transformation, identification process and in designing controller law which is discussed in detail in the following sections.

In the second section the embedding properties, Poincare map of the embedded chaotic attractor and controlling the original system have been explained in detail. In the third section, the procedure has been examined on Duffing and chaotic pendulum equations and the results have been reported. Finally, in the last section, a brief conclusion has been presented.

2. Problem statement

The problem which has been solved in this article is controlling a continuous time chaotic system with an unknown governing

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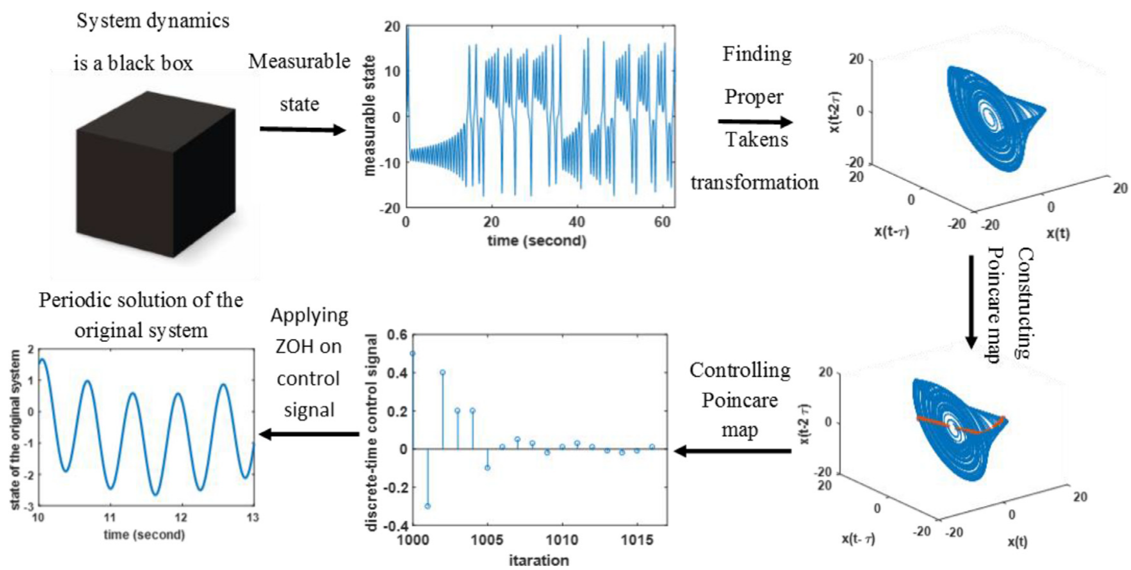

Fig. 1. Steps toward controlling the original system.

equation when at least one of the states of the system is measurable. We construct a new chaotic attractor by using time series of the measurable state through Takens embedding theory. After generating a new chaotic attractor, we made the Poincare map of the reconstructed chaotic system. Then we found the unstable fixed points of the obtained Poincare map and finally after designing a control law for the Poincare map we apply a proper control signal to the original system. Fig. 1 displays different steps which should be taken to control the chaotic system.

2.1. Embedding new attractor

The disability to measure all of the states and being blind to the dynamics of the system urges us to use Takens embedding theory. This theory uses time series of a single measurable state to reconstruct new chaotic attractor which preserves differential information of the unknown chaotic attractor. Unfortunately, Takens' theory assumes that there are infinite noise-free data. But in real systems, we have limited and noisy data [9,10]. So, special consideration should be taken into account in choosing the embedding properties. If there is an unknown governing like Eq. (1), with unknown k and g_i on the original system, then Takens transformation, ϕ is defined as Eq. (2):

$$\begin{cases} \dot{x}_1(t) = g_1(x_1, \ldots, x_k) \\ \dot{x}_2(t) = g_2(x_1, \ldots, x_k) \\ \vdots \\ \dot{x}_k(t) = g_k(x_1, \ldots, x_k) \end{cases} \tag{1}$$

$$\mathbf{x}(j) : \begin{bmatrix} x_1(t) \\ x_2(t) \\ \vdots \\ x_k(t) \end{bmatrix} \xrightarrow{\phi} \begin{bmatrix} x_j(t) \\ x_j(t-\tau) \\ \vdots \\ x_j(t-n\tau) \end{bmatrix} \tag{2}$$

In Eq. (2) the first vector consists of the states of the original system and the second vector shows the states of the reconstructed system where x_j is the measurable state of the original system. The other parameters τ and n are pretty much crucial in quality of the Takens transformation. Delayed time 'τ' is usually set to be unity in discrete time systems. But finding proper τ is one of the most important parts of the finding correct transformation in continuous time systems. In this paper we do not concentrate on finding these parameters (τ, n) and we have used studies done before on this topic [11–13].

2.2. Poincare map of the reconstructed system

To control the original system, first, we have to control the reconstructed system. The governing equation on the reconstructed system is as Eq. (3),

$$\begin{cases} \dot{x}_j(t) = f(x_j(t), x_j(t-\tau), \ldots, x_j(t-n\tau)) \\ \dot{x}_j(t-\tau) = f(x_j(t-\tau), x_j(t-2\tau), \ldots, x_j(t-(n+1)\tau)) \\ \vdots \\ \dot{x}_j(t-n\tau) = f(x_j(t-n\tau), x_j(t-(n+1)\tau), \ldots, x_j(t-2n\tau)) \end{cases} \tag{3}$$

Eq. (3) is a delayed differential equation and identification of this equation is pretty much complex. To skip this identification process we have employed Poincare map of the system.

2.3. Controlling the Poincare map of the reconstructed system

Skipping identification of 'f' function in Eq. (3) leads us to use the Poincare map of the reconstructed system for the controlling purposes. To design a linear controller for the Poincare map, a linearized Poincare map defined near the unstable fixed point (UFP) of the reconstructed system is utilized. So the Jacobian matrix of the Poincare map calculated on the UFP should be found numerically. The linearization of Poincare map on the UFP, denoted by $\mathbf{X_f}$, results in the following equation,

$$\left(\begin{bmatrix} y_1(k+1) \\ y_2(k+1) \\ \vdots \\ y_p(k+1) \end{bmatrix} - \mathbf{X_f} \right) \approx [\mathbf{J}] \left(\begin{bmatrix} y_1(k) \\ y_2(k) \\ \vdots \\ y_p(k) \end{bmatrix} - \mathbf{X_f} \right) + \mathbf{u} \tag{4}$$

where $[\mathbf{J}]$ is the Jacobian matrix, and $\mathbf{u}$ is the control signal which is designed by linear feedback method. 'p' is the dimension of the Poincare map, and $\mathbf{y}(k) = [y_1(k), \ldots, y_p(k)]^T$ is the state vector of the system defined by the Poincare map. Note that the linear approximation (Eq. (4)) is valid only in a small vicinity of the UFP so the corresponding control law should be applied when the system is close enough to the UFP. The form of the control signal is