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Rogue waves for the coupled variable-coefficient fourth-order nonlinear Schrödinger equations in an inhomogeneous optical fiber

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ABSTRACT

In this paper, investigation is made on the coupled variable-coefficient fourth-order nonlinear Schrödinger equations, which describe the simultaneous propagation of optical pulses in an inhomogeneous optical fiber. Via the generalized Darboux transformation, the first- and second-order rogue wave solutions are constructed. Based on such solutions, effects of the group velocity dispersion coefficient and the fourth-order dispersion coefficient on the rogue waves are graphically analyzed. The first-order rogue waves with the eye-shaped distribution, the interactions between the first-order rogue waves with solitons, and the second-order rogue waves with one highest peak and with the triangular structure are displayed. When the value of the group velocity dispersion coefficient or the fourth-order dispersion increases, range of the first-order rogue wave increases. Composite rogue waves are obtained, where the temporal separation between two adjacent rogue waves can be changed if we adjust the group velocity dispersion coefficient and fourth-order dispersion coefficient. Periodic rogue waves are presented. Periods of such rogue waves decrease with the periods of the group-velocity dispersion and fourth-order dispersion coefficient decreasing.

1. Introduction

Rogue waves, which have much larger amplitudes than the average wave crests around them [1], have been found to unpredictably appear on the ocean surfaces [2,3]. Apart from the ocean, rogue waves have been found in the optical fibers [4–8]. In such photonic crystal fibers as the mode-locked fiber lasers, fiber Raman amplifiers and whispering-gallery-mode resonators, experimental and theoretical investigations on the rogue waves have been done [5–7]. The same as the solitons and breathers [9,10], rogue waves have been seen to be modeled by the nonlinear Schrödinger (NLS) equation related to the group velocity dispersion (GVD) and self-phase modulation, which describes the evolution of a weakly nonlinear wave packet in the deep water and a picosecond optical pulse propagation in the nonlinear optical fiber [4,11,12].

Coupled NLS-type equations have been seen to describe the propagation of the multiple optical waves in the nonlinear media [13–16]. When the inhomogeneous optical fibers are involved, it has been thought that the variable-coefficient NLS equations are more precise than their constant-coefficient versions [17]. Rogue waves for the coupled variable-coefficient NLS equations and coupled variable-coefficient Hirota equations have been investigated [18,19]. For the simultaneous propagation of nonlinear waves in the inhomogeneous optical fiber, people have reported the following coupled variable-coefficient fourth-order NLS equations [20,21]:

$$
\begin{aligned}
& iq_{j,t}+\sigma(t)q_{j,xx}+\mu(t)q_j\sum_{l=1}^{2}|q_l|^2+\gamma_1(t)q_{j,xxxx}+\gamma_2(t)q_j\sum_{l=1}^{2}|q_{l,x}|^2\\
& +\gamma_3(t)q_{j,x}\sum_{l=1}^{2}q_l q_{l,x}^{*}+\gamma_4(t)q_{j,x}\sum_{l=1}^{2}q_l^{*}q_{l,x}+\gamma_5(t)q_{j,xx}\sum_{l=1}^{2}|q_l|^2\\
& +\gamma_6(t)q_j\sum_{l=1}^{2}q_l^{*}q_{l,xx}+\gamma_7(t)q_j\sum_{l=1}^{2}q_l q_{l,xx}^{*}+\gamma_8(t)q_j\Big(\sum_{l=1}^{2}|q_l|^2\Big)^2\\
& =0,\quad (j=1,\,2),
\end{aligned}
\tag{1}
$$

where $q_1(x,\,t)$ and $q_2(x,\,t)$ denote the complex envelopes of two field polarization components, the subscripts x and t represent the partial derivatives with respect to the normalized propagation distance and retarded time respectively, $\sigma(t)$ denotes the GVD coefficient, $\mu(t)$ is related to the self-phase modulation coefficient, $\gamma_1(t)$ is the fourth-order dispersion coefficient, $\gamma_\alpha(t)$'s $(\alpha=2,\cdots,7)$ denote the cubic nonlinear coefficients, $\gamma_8(t)$ is the quintic nonlinear coefficient, and "$*$" represents the complex conjugate. Bound-state solitons and interactions between the two solitons for

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Eqs. (1) have been discussed via the Hirota method [20]. Lax pair and dark–dark solitons for Eqs. (1) have been reported via the binary Darboux transformation [21].

Different from those in Ref. [21], we will construct the Darboux transformation (DT) and generalized DT (GDT) for Eqs. (1) in Section 2. In Section 3, the first- and second-order rogue wave solutions for Eqs. (1) will be acquired via the GDT. Based on such solutions, interactions between the rogue waves and solitons, composite and periodic rogue waves will be graphically analyzed in Section 4. Our conclusions will be given in Section 5.

2. DT and GDT for Eqs. (1)

According to the Ablowitz–Kaup–Newel–Segur (AKNS) system [22], Lax pair for Eqs. (1) can be cast into the following form [21]:

$$\Phi_x = U\Phi, \qquad \Phi_t = V\Phi, \tag{2}$$

where $\Phi = (\Phi_1, \Phi_2, \Phi_3)^T$ is the vector eigenfunction, the superscript T denotes the transpose for a vector, Φ_1, Φ_2 and Φ_3 are the scalar eigenfunctions with respect to λ, x and t, and matrices U and V are written in the forms of

$$U = \begin{pmatrix} -i\lambda & q_1 & q_2 \\ -q_1^* & i\lambda & 0 \\ -q_2^* & 0 & i\lambda \end{pmatrix}, \qquad V = \begin{pmatrix} \mathrm{v}_{11} & \mathrm{v}_{12} & \mathrm{v}_{13} \\ \mathrm{v}_{21} & \mathrm{v}_{22} & \mathrm{v}_{23} \\ \mathrm{v}_{31} & \mathrm{v}_{32} & \mathrm{v}_{33} \end{pmatrix}, \tag{3}$$

with λ being the spectral parameter, and

$$\begin{aligned}
\mathrm{v}_{11} &= 8i\gamma_1(t)\lambda^4 - 2i\Big[\sigma(t) + 2\gamma_1(t)\sum_{l=1}^{2}|q_l|^2\Big]\lambda^2 + 2\gamma_1(t) \\
&\quad \times \sum_{l=1}^{2}(q_l^* q_{l,x} - q_l q_{l,x}^*)\lambda - i\gamma_1(t)\Big[\sum_{l=1}^{2}|q_{l,x}|^2 \\
&\quad - \sum_{l=1}^{2}(q_l^* q_{l,xx} + q_l q_{l,xx}^*) - 3\Big(\sum_{l=1}^{2}|q_l|^2\Big)^2\Big] + i\sigma(t)\sum_{l=1}^{2}|q_l|^2, \\
\mathrm{v}_{1,s} &= -8\gamma_1(t)q_{s-1}\lambda^3 - 4i\gamma_1(t)q_{s-1,x}\lambda^2 + 2\Big[\sigma(t)q_{s-1} \\
&\quad + \gamma_1(t)q_{s-1,xx} + 2\gamma_1(t)q_{s-1}\sum_{l=1}^{2}|q_l|^2\Big]\lambda + i\gamma_1(t)\Big[q_{s-1,xxx} \\
&\quad + 3q_{s-1,x}\sum_{l=1}^{2}|q_l|^2 + 3q_{s-1,x}\sum_{l=1}^{2}q_l^* q_{l,x}\Big] + i\sigma(t)q_{s-1,x}, \\
\mathrm{v}_{s,s} &= -8i\gamma_1(t)\lambda^4 + 2i\Big[\sigma(t) + 2\gamma_1(t)|q_{s-1}|^2\Big]\lambda^2 + 2\gamma_1(t)(q_{s-1}q_{s-1,x}^* \\
&\quad - q_{s-1}^* q_{s-1,x})\lambda - 3i\gamma_1(t)|q_{s-1}|^2\sum_{l=1}^{2}|q_l|^2 + i\gamma_1(t)\Big[|q_{s-1,x}|^2 \\
&\quad - q_{s-1}q_{s-1,xx}^* - q_{s-1}^* q_{s-1,xx}\Big] - i\sigma(t)|q_{s-1}|^2, \\
\mathrm{v}_{23} &= 4i\gamma_1(t)q_1^* q_2\lambda^2 + 2\gamma_1(t)(q_{1,x}^* q_2 - q_1^* q_{2,x})\lambda \\
&\quad - 3i\gamma_1(t)q_1^* q_2\sum_{l=1}^{2}|q_l|^2 + i\gamma_1(t)(q_{2,x}q_{1,x}^* - q_1^* q_{2,xx} - q_2 q_{1,xx}^*) \\
&\quad - i\sigma(t)q_1^* q_2, \\
\mathrm{v}_{32} &= -\mathrm{v}_{23}^*, \quad \mathrm{v}_{s,1} = -\mathrm{v}_{1,s}^*, \quad (s = 2, 3).
\end{aligned}$$

It can be checked that the compatibility condition $U_t - V_x + UV - VU = 0$ is equivalent to Eqs. (1) under the constraints

$$\mu(t) = 2\sigma(t), \ \gamma_2(t) = 2\gamma_1(t), \ \gamma_3(t) = 2\gamma_1(t), \quad \gamma_4(t) = 6\gamma_1(t),$$
$$\gamma_5(t) = 4\gamma_1(t), \ \gamma_6(t) = 4\gamma_1(t), \ \gamma_7(t) = 2\gamma_1(t), \quad \gamma_8(t) = 6\gamma_1(t).$$

In the following, we will construct the DT and generalized DT for Eqs. (1). Under the transformation,

$$\Phi^{[1]} = M^{[1]}\Phi, \tag{4}$$

Lax Pair (2) can be transformed into

$$\Phi_x^{[1]} = U^{[1]}\Phi^{[1]}, \quad U^{[1]} = M_x^{[1]}(M^{[1]})^{-1} + M^{[1]}U(M^{[1]})^{-1}, \tag{5a}$$

$$\Phi_t^{[1]} = V^{[1]}\Phi^{[1]}, \quad V^{[1]} = M_t^{[1]}(M^{[1]})^{-1} + M^{[1]}V(M^{[1]})^{-1}, \tag{5b}$$

where $U^{[1]}$ or $V^{[1]}$ has the same form as that of U or V with q_j and q_j^* replaced by $q_j^{[1]}$ and $q_j^{[1]*}$, respectively, $[k]$ $(k = 0, 1, 2, 3, \cdots)$ represents the kth iteration, $\Phi^{[1]}$ is a 3×1 matrix, $M^{[1]}$, $U^{[1]}$ and $V^{[1]}$ are all the 3×3 nonsingular matrices, and "−1" denotes the matrix inverse. From Expressions (5), we can obtain

$$U_t^{[1]} - V_x^{[1]} + U^{[1]}V^{[1]} - V^{[1]}U^{[1]} = M^{[1]}(U_t - V_x + UV - VU)(M^{[1]})^{-1}, \tag{6}$$

which implies that Lax Pair (2) keeps invariant under Transformation (4). We need to acquire a matrix $M^{[1]}$ such that $U^{[1]}$ or $V^{[1]}$, respectively, possesses the same form of U or V, while (q_1, q_2) in U or V is mapped into $(q_1^{[1]}, q_2^{[2]})$ in $U^{[1]}$ or $V^{[1]}$.

The DT matrix $M^{[1]}$ for Eqs. (1) has the form of [23]

$$M^{[1]} = \lambda I - H\Lambda H^{-1}, \tag{7}$$

with

$$H = \begin{pmatrix} \varphi_{11} & -\varphi_{21}^* & -\varphi_{31}^* \\ \varphi_{21} & \varphi_{11}^* & 0 \\ \varphi_{31} & 0 & \varphi_{11}^* \end{pmatrix}, \quad \Lambda = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_1^* & 0 \\ 0 & 0 & \lambda_1^* \end{pmatrix}, \tag{8}$$

where λ_1 is a given parameter to construct the first-order DT, I is a 3×3 identity matrix, H is the 3×3 nonsingular matrix, the eigenfunction $(\varphi_{11}, \varphi_{21}, \varphi_{31})^T$ is the vector solution of Lax Pair (2) at $\lambda = \lambda_1$, φ_{11}, φ_{21} and φ_{31} are the scalar functions with respect to x and t. Therefore, the first-order DT for Eqs. (1) can be indicated as

$$q_1^{[1]} = q_1^{[0]} - 2i\frac{(\lambda_1 - \lambda_1^*)\varphi_{11}\varphi_{21}^*}{|\varphi_{11}|^2 + |\varphi_{21}|^2 + |\varphi_{31}|^2}, \tag{9a}$$

$$q_2^{[1]} = q_2^{[0]} - 2i\frac{(\lambda_1 - \lambda_1^*)\varphi_{11}\varphi_{31}^*}{|\varphi_{11}|^2 + |\varphi_{21}|^2 + |\varphi_{31}|^2}, \tag{9b}$$

where $q_1^{[0]}$ and $q_2^{[0]}$ are the seed solutions for Eqs. (1), $q_1^{[1]}$ and $q_2^{[1]}$ denote the first-order DT solutions for Eqs. (1). Similarly, the second-order DT for Eqs. (1) can be obtained as

$$q_1^{[2]} = q_1^{[1]} - 2i\frac{(\lambda_2 - \lambda_2^*)\varphi_{12}\varphi_{22}^*}{|\varphi_{12}|^2 + |\varphi_{22}|^2 + |\varphi_{32}|^2}, \tag{10a}$$

$$q_2^{[2]} = q_2^{[1]} - 2i\frac{(\lambda_2 - \lambda_2^*)\varphi_{12}\varphi_{32}^*}{|\varphi_{12}|^2 + |\varphi_{22}|^2 + |\varphi_{32}|^2}, \tag{10b}$$

where λ_2 is a given parameter to construct the second-order DT, $\varphi_{m,2}$'s $(m = 1, 2, 3)$ are all the complex scalar functions with respect to x and t, $(\varphi_{12}, \varphi_{22}, \varphi_{32})^T$ is the vector solution of Lax Pair (2) at $\lambda = \lambda_2$, and $q_1^{[2]}$ and $q_2^{[2]}$ denote the second-order DT solutions for Eqs. (1).

Based on DT (9) and (10), we derive the generalized DT for Eqs. (1). Assume that $\Phi(\lambda_1 + \delta)$ is a solution for Lax Pair (2) with $\lambda = \lambda_1 + \delta$ and $q_1 = q_1^{[0]}$ and $q_2 = q_2^{[0]}$, where δ is a small parameter. Expanding $\Phi(\lambda_1 + \delta)$ into the Taylor series at $\lambda = \lambda_1$, we can acquire

$$\Phi(\lambda_1 + \delta) = \Phi_0 + \Phi_1\delta + \Phi_2\delta^2 + \cdots + \Phi_\alpha\delta^\alpha + \cdots, \tag{11}$$