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Localization in one-dimensional tight-binding model with chaotic binary sequences

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ABSTRACT

We have numerically investigated localization properties in the one-dimensional tight-binding model with chaotic binary on-site energy sequences generated by a modified Bernoulli map with the stationary-nonstationary chaotic transition (SNCT). The energy sequences in question might be characterized by their correlation parameter B and the potential strength W. The quantum states resulting from such sequences have been characterized in the two ways: Lyapunov exponent at band centre and the dynamics of the initially localized wavepacket. Specifically, the $B-$dependence of the relevant Lyapunov exponent's decay is changing from linear to exponential one around the SNCT ($B \simeq 2$). Moreover, here we show that even in the nonstationary regime, mean square displacement (MSD) of the wavepacket is noticeably suppressed in the long-time limit (dynamical localization). The $B-$dependence of the dynamical localization lengths determined by the MSD exhibits a clear change in the functional behaviour around SNCT, and its rapid increase gets much more moderate one for $B \geq 2$. Moreover we show that the localization dynamics for $B > 3/2$ deviates from the one-parameter scaling of the localization in the transient region.

1. Introduction

It has been known that in one-dimensional disordered systems (1DDS) with uncorrelated on-site disorder all eigenstates are exponentially localized [1–3]. Still, for the 1D tight-binding model with potential sequences generated by Fourier filtering method (FFM) it has been known that correlations arising in the on-site potential delocalize the eigenstates and induce localization-delocalization transition (LDT) [4–16]. Indeed, the potential sequences involved ought to have long-range correlation with power spectrum $S(f) \sim 1/f^{\alpha}$ $(f << 1, \alpha \geq 2)$, where f denotes frequency and α is spectrum index. The potential sequence is non-stationary when the total power $\int_0^{\infty} S(f)df$ is divergent.

Noteworthy, the results do not contradict the Kotani theory of the localization stating that if the stationary random potential is non-deterministic, absolutely continuous spectrum is absent. The stationarity is a sufficient condition for the absence of absolutely continuous spectrum [17]. On the other hand, the potential sequence characterized by the power spectrum with the exponent $\alpha > 1$ would be nonstationary. Further, most recent numerical studies show that the sequences with the power-law spectrum generated by Weierstrass function with fractal dimension $1 < D < 2$ induce the LDT [18–22].

There are systematic numerical studies for the above-mentioned 1DDS models with a potential to take continuous value like in the Anderson model. Whereas uncorrelated random model (e.g., Bernoulli Anderson model) with discretized values are well-known to show specific localization phenomena, the number of studies of localization and delocalization with the correlated binary potential are still few [23–30]. E.g., among the latter examples the following one should be mentioned. There is a study of delocalization in binary "0" and "1" system and the sparse potential which takes different values for prime sites only [26]. It is in such a case that the very "sparse" model should also naturally be "nonstationary". Remarkably, the existence of the LDT due to the potential intensity has also been demonstrated in the sparse impurity distance model [31].

Furthermore, a number of works also have been published on localized and delocalized phenomena in 1DDS with deterministic correlated sequence generated by chaotic map [23,32–35]. In our earlier papers, we also numerically investigated the localization and delocalization phenomena of binary random systems with long-range correlation by the modified Bernoulli map with stationary-nonstationary chaotic transition (SNCT) [23]. We shall refer to such a system MB system in the following [36–38]. The sequence becomes asymptotic non-stationary chaos for $\alpha > 1$. In the MB system, it is possible to create the potential sequence that changes the property from short-range correlations includ-

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ing $\delta-$correlations to long-range correlation with a gentle change of the correlation parameter B. Meanwhile, studying in detail the modalities of the localization events, especially in the situations, where transitions from stationary $(3/2 < B < 2)$ to nonstationary regime $(B > 2)$ regimen takes place in the binary correlated 1DDS has still not been enough. In particular, wavepacket dynamics in the nonstationary potential has hardly been investigated [39–43]. In this paper, we use the long-range correlated MB system having the binary potential sequence with taking either one of $-W$ or W, like in our previous papers. We aim at reporting the characteristic $B/W-$dependences of the Lyapunov exponent, the normalized localization length (NLL), and the quantum diffusion of the initially localized wavepacket around SNCT in binary correlated disordered systems.

Generally, the statistical properties of the deterministic chaotic sequence such as the higher order correlations is quite different from the probabilistic one even though it have the same power spectrum, due to the difference of the ergodic measure. Quantitative transition of the Lyapunov exponent and dynamical localization length around SNCT ($B = 2$ or $\alpha = 1$) have not been reported in the 1DDS with the probabilistic sequence. It is the purpose of this report to investigate the qualitative changes in MB system that the statistical property can be finely adjusted by changing the parameter B.

This paper is organized as follows. In the next section, we shall briefly introduce the modified Bernoulli model. In Section 3 we report about the global behaviour of the $B-$dependence and $N-$dependence of Lyapunov exponent and the NLL at band centre by the numerical calculation. While the Lyapunov exponent is positive throughout all the B regions studied here, the Lyapunov exponent decreases linearly for $B < 2$, but decays exponentially for $B > 2$. As a result, the quantum states get delocalized $(\gamma_N \to 0)$ with $B \to \infty$. In Section 4, we report on the dynamical localization phenomena in the system. We find that the MSD is finite and dynamically localized in $t \to \infty$ even if the correlation parameter changes from stationary regime $B < 2$ with power-law decay of the correlation to nonstationary regime ($B \geqq 2$). Its dynamical localization length (DLL) increases with the correlation parameter B, but the $B-$dependence changes from a relatively rapid increase to a more moderate one around SNCT($B \simeq 2$). The one-parameter scaling based on the localization length has large fluctuation in the transient region from ballistic motion to localization for $B > 3/2$. The summary and discussion are presented in the last section. Appendix shows the sample fluctuation including nonstationary regime.

2. Model

We consider the one-dimensional tight-binding Hamiltonian describing single-particle electronic states as

$$H = \sum_{n=1}^{N} W v(n) c_n^\dagger c_n + \sum_{n=1}^{N-1} c_n^\dagger c_{n+1} + H.C., \quad (1)$$

where $c_n^\dagger (c_n)$ is the creation (annihilation) operator for an electron at site n. The $\{v_n\}_{n=0}^N$ and W are the disordered on-site energy sequence and the strength, respectively. The amplitude of the quantum state $|\Phi>$ is given by $\phi(n) \equiv \langle \Phi | c_n^\dagger c_n | \Phi \rangle$ in the site representation. To model the correlated disorder potential for $v_n (n \le N)$ in Eq. (1), we use the modified Bernoulli map;

$$X_{n+1} = \begin{cases} X_n + 2^{B-1}(1-2b)X_n^B + b & (0 \le X_n < 1/2) \\ X_n - 2^{B-1}(1-2b)(1-X_n)^B - b & (1/2 \le X_n \le 1), \end{cases} \quad (2)$$

where B is a bifurcation parameter which controls the correlation of the sequence. b stands for the small perturbation which is set $b = 10^{-13}$ in this paper. The map has been introduced to investigate the basic property of the intermittent chaos by Aizawa et al. [36].

The sequence is stationary for $B < 2$ and nonstationary for $B \ge 2$. The stationary property is recovered by the perturbation though the essential property remains invariant for a long time $n < n_b$, where $n_b \simeq (b)^{(1-B)/B}$ [36]. Introduction of the b is useful for asymptotically examining the nonstationary region numerically from the stationary region, but numerical result with the value $b = 10^{-13}$ in this report is the same as the result when $b = 0$.

We use the course-grained binary sequence $\{v_n\}$ by the following rule:

$$\begin{cases} 0 \le X_n < 1/2 & \to v_n = -1 \\ 1/2 \le X_n < 1 & \to v_n = 1. \end{cases} \quad (3)$$

Accordingly, the statistical property of the binary sequence can be characterized by changing the correlation parameter B. The following properties, for example, are analytically and numerically derived. In the stationary regime $(3/2 < B < 2)$ the correlation function of the symbolic sequence decreases obeying the inverse-power law with the long-range correlation for large n [36],

$$C(n) \equiv < v_{n_0+n} v_{n_0} > \sim n^{-\frac{2-B}{B-1}} (n \gg 1). \quad (4)$$

The correlation shows the critical decay $C(n) \sim 1/n$ at $B = 3/2$. In the nonstationary regime $(B \ge 2)$ the correlation decays as,

$$C(n) \simeq 1 - \frac{2}{B}\left(\frac{n}{n_b}\right)^{\frac{B-2}{B-1}}, \quad (5)$$

for $n \le n_b$. The power spectrum $S(f) = \frac{1}{N}|\sum_{n=0}^{N} e^{-i2\pi fn/N}|^2$ $(f = 0, 1, 2, \ldots, N-1)$ in the low frequency limit behaves

$$S(f) \sim \begin{cases} f^0 & 1 \le B < 3/2 \\ f^{-\alpha} & 3/2 \le B \le \infty, \end{cases} \quad (6)$$

in the thermodynamic limit $(N \to \infty)$, where

$$\alpha \simeq \frac{2B-3}{B-1}. \quad (7)$$

That is, the stationary sequence changes to nonstationary one with $S(f) \sim 1/f$ around $B \simeq 2$. It is suggested that in FFM model and Weierstrass model with long-range correlation LDT appear in a case with $\alpha \simeq 2$. Note that if $B \to \infty$, then $S(f) \sim f^{-2}$ as shown in Fig. 1. Note that these are true for $B = 0$, and theoretical cut-off time and cut-off frequency are given by $1 \le n \le n_b$ and $1/n_b < f < <1$, respectively, for $b > 0$. The initial ensemble is taken based on the equilibrium renewal process. It has been shown that the correlation function depends on the initial ensemble [44,45].

Still, the localization property of 1DDS around $\alpha \simeq 1$ have not yet been studied. In the present paper, we investigate the change of the quantum states around the SNCT of the sequenece. It has already been reported that this property of the sequence strongly affects the statistical nature of the Lyapunov exponents of the electronic wave functions [23].

Moreover, the binary sequence $\{v_n\}$ can be recast as $\{(m_0, \sigma), (m_1, -\sigma), (m_2, \sigma), (m_3, -\sigma) \ldots\}$. Here (m_k, σ) stands for the m_k times iterating of one and the same symbol σ, where σ represents -1 or 1. The sequence is uniquely determined by the cluster size distribution $P(m)$ for the number m of iterations in the pure sequence (m, σ), which is independent of the value of the symbol. Hence, the time interval m between successive renewal events is a random variable, whose probability density function $P(m)$ as $b = 0$:

$$P(m) \sim m^{-\beta}, \quad (8)$$