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The route to synchrony via drum head mode and mixed oscillatory state in star coupled Hindmarsh–Rose neural network

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ABSTRACT

Network of coupled oscillators exhibit different types of spatiotemporal patterns. We report that as the coupling strength increases the unidirectionally coupled Hindmarsh–Rose neuron star network will synchronize. The condition for synchronization has been evaluated using Lyapunov function method. We also discuss the dynamics of the system in the presence of controllers. The control input generate interesting behaviors which consist of clusters of spatially coherent domains depending on the coupling strength. Drum head mode, mixed oscillatory state, desynchrony, and multi cluster states are formed and cluster reduction takes place before settling to complete synchrony. The evolution of a perfectly synchronized state via drum head mode, mixed oscillatory state, and clusters from a desynchronized state is reported for the first time. The parameter values which lead to stable cluster formation is also discussed. Our results suggest that in the presence of controllers the common oscillator in the star network behaves as a driver and generates the transitions and cluster formation acts as a precursor to complete synchrony in Hindmarsh–Rose model with unidirectional star coupling.

1. Introduction

The cooperative behavior of complex systems have many applications in dynamical system theory. Synchronization phenomena is such a cooperative behavior commonly exhibited by physical, chemical, and biological systems [1,2]. Christiaan Huygens was the first one to observe the synchronization of two coupled pendulum clocks in 16th century [3]. Numerous experimental and theoretical works describing the dynamics of neural network and its application to brain science have been reported in literature [4–10]. The irregular behavior of individual neurons can be controlled by connecting them using suitable coupling topologies which may lead to their synchronization and rhythmic activity [11,12].

There has also been some considerable interest on the various spatiotemporal patterns shown by coupled oscillator network. The role of noise in chaotic systems and non chaotic systems have been reported [13–16]. The coherence resonance in the presence of noise has also been studied recently [17]. Coupling topologies like all to all coupling (global), nearest neighbor coupling (lattice), and nonlocal coupling of chaotic maps and Kuramoto phase oscillators were extensively studied [18–22]. When the coupling strength is increased the globally coupled chaotic maps show a transition from desynchrony to synchrony via the formation of clusters [23,24], i.e., all oscillators within one cluster fire in exact synchrony. Synchronous clusters are obtained when oscillators synchronize with the members in the same group and no synchronization between the groups. Cluster formation in neural network is a mesoscale phenomenon which includes the cooperative rhythms of neuronal subpopulations whereas synchronization is a macroscale phenomenon with large scale patterns of activity. Cluster synchrony is highly dependent on the structure and symmetries of the network [25–28]. Belykh et al. have reported the cluster synchronization of nonlocally coupled oscillators [29]. Clustering in a mean-field coupled Rosenzweig–MacArthur model has been reported by Arumugam et al. [30]. In star coupled oscillators Pecora and Carroll have reported an important desynchronization bifurcation in which the nodes on the spokes show synchrony while the hub exhibits different dynamics termed as drum head mode (DHM) [31–33]. An interesting novel type of dynamics called mixed oscillatory state (MOS) has been identified recently in Hindmarsh–Rose (HR) neurons with nonlocal coupling [34].

In this work we analyze the presence of DHM, MOS, cluster formation, and synchronization of star coupled HR neural network with electrical coupling. We have chosen star unidirectional cou-

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pling topology in which all oscillators on the spoke of the star are connected to one common oscillator in the network thereby providing minimum number of connections within the system. The common oscillator is called the hub, and the oscillators on the spoke of the star are termed as nodes.

The experimental realization of spoke hub distribution paradigm and the importance of unidirectional coupling in human brain are widely investigated. Identifying the hub regions and their pivotal role in the coordination of brain activities is of great importance. Hubs can be classified as provincial and connector hubs. The provincial hub occupies at the middle of a single functional cluster and the connector hub links the multiple functional parts like visual and sensorimotor areas of brain [35]. The contribution of individual brain regions within the cerebral cortex to overall brain activity has been established in cat and macaque [35]. The electroencephalography (EEG), magnetoencephalography (MEG), and functional magnetic resonance imaging (fMRI) studies have also revealed the functional role of individual brain regions in integrating the over all information processing [36]. The recent studies on the synchronization between the heart signals and EEG frequency bands revealed that a strong unidirectional coupling from brain to heart exists during all sleep stages (stages 1–4) [37]. The EEG and MEG analysis on the cortical activation patterns show that, a remarkable unidirectional coupling from contralateral motor cortex to muscles exist in the swing leg during treadmill stereotyped walking [38]. The breathing and heartbeat generators acts like two weakly coupled oscillators and their synchronization is enhanced by an uncorrelated noise from brain. This coupling has unidirectional nature, with a coupling direction from breathing to heart beat [39]. In communication systems this type of network structure is very important because the hub acts as a driver which controls the entire network. In computer networks, the hub in star topology can be considered as a server and the nodes as the clients. Here we mainly discuss the role of hub in providing equal coupling to the nodes and its driving mechanism to form DHM, MOS, and clustered states.

The paper is arranged as follows. Section 2 describes the HR model and the collective behavior of star coupled HR network by varying the values of coupling strength and the parameter describing the activation–inactivation dynamics of fast ion channel. The control inputs for synchronizing the system are evaluated using Lyapunov function method in Section 3. DHM, MOS, cluster formation, and synchronization are also explained in Section 3. Section 4 concludes the study.

2. The model

2.1. HR model

We start with HR model, described by the following set of differential equation,

$$\dot{x} = y + ax^2 - x^3 - z + I,$$
$$\dot{y} = 1 - bx^2 - y, \tag{1}$$
$$\dot{z} = r(R(x - x_e) - z),$$

where x denotes the membrane potential. y is the recovery variable representing the rate of change of fast current of K^+ or Na^+ ions and z denotes the adaptation variable which capture the slower dynamics of other ion channels (e.g., Ca^+). The parameters a and b denote activation and inactivation of the fast ion channel whereas R and x_e describe activation and inactivation of the slow ion channel. The speed of variation of z is controlled by the parameter r [40–42]. The parameter I represents the external current that reaches the neuron. The model describes the firing behavior of neurons and can be decomposed in to a slow fast system and the slow oscillations of z drives the fast subsystem (x, y) through periods of quiescent and oscillatory behavior [29]. If the parameters are chosen as $a = 3.0,\ b = 5.0,\ R = 4.0,\ r = 0.006$ and $x_e = -1.61$ [43], the model exhibits resting state, regular spiking, regular bursting and chaotic bursting by varying I. For I in range $0 < I < 1.3$, the trajectories are stabilized. When $I = 1.39$, periodic spiking behavior starts and for $I = 1.7$, the behavior of the system changes from periodic spiking to periodic bursting. As I reaches 3.1 unstable nature of the system grows and it becomes chaotic [43]. In our simulations $I = 3.1$ is chosen to ensure the chaotic bursting dynamics of isolated units.

2.2. Star coupled HR model and synchronization

Now consider a network of 'N' HR neurons with electrical coupling in star unidirectional connection topology. The Nth oscillator is chosen as the hub and oscillators 1 to $N - 1$ are arranged on the spoke of the star [44]. The state equations are

$$\dot{x}_i = y_i + ax_i^2 - x_i^3 - z_i + I + g(x_N - x_i),$$
$$\dot{y}_i = 1 - bx_i^2 - y_i, \tag{2}$$
$$\dot{z}_i = r(R(x_i - x_e) - z_i), \qquad i = 1, 2, \ldots, N.$$

Due to the extremely small time scale associated with the transmission of nerve impulse at the synaptic junction, we consider instantaneous coupling to model the system. The coupling between the oscillators are established through $g(x_N - x_i)$ in which g represents the coupling strength between the oscillators. The coupling topology used in Eq. (2) is star unidirectional, where x_N is the membrane potential of Nth neuron (hub) to which all others are connected. As a result of unidirectional coupling the hub exhibits chaotic bursting irrespective of the value of g.

Primarily we have analyzed the collective dynamics of star coupled network with increase in coupling strength. For $0 < g < 0.85$ the oscillators show incoherent behavior, i.e., desynchrony prevails as a result of weak coupling as indicated in Fig. 1(a). When the coupling strength increases, i.e., $g \geq 0.85$ all neurons in the network show synchronous behavior as shown in Fig. 1(b) with chaotic bursting time series. The star coupled HR neural network with electrical coupling shows synchronization as the coupling strength is increased.

2.3. Stability of synchronization

The stability of synchronization can be quantified using the master stability approach developed by Pecora and Carroll [32]. The synchronization is stable if the master stability function is negative at each of the transverse eigenvalues. The analytical expressions for estimating the synchronization threshold for diffusively coupled continuous and discrete time chaotic systems have been reported [45].

On the completely stable synchronization manifold, the differences between neural oscillator coordinates $x_{i_\perp} = x_N - x_i,\ y_{i_\perp} = y_N - y_i$ and $z_{i_\perp} = z_N - z_i$ vanish in the limit of $t \longrightarrow \infty$ and there exist a synchronous solution $\zeta_1(t) = \zeta_2(t) = \cdots = \zeta_N(t)$, where $\zeta_i(t) = (x_i, y_i, z_i)$. The stability equations for perturbations transverse to the synchronization manifold is,

$$\dot{x}_{i_\perp} = y_{i_\perp} + 2axx_{i_\perp} - 3x^2x_{i_\perp} - z_{i_\perp} - gx_{i_\perp},$$
$$\dot{y}_{i_\perp} = -2bxx_{i_\perp} - y_{i_\perp}, \tag{3}$$