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Fixed-time synchronization of hybrid coupled networks with time-varying delays

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ABSTRACT

In this paper, we study the fixed-time synchronization of hybrid coupled networks, which have only one transmittal delay in the delayed coupling terms. The settling time of fixed-time synchronization can be adjusted to some desired values in advance regardless of the initial conditions. By constructing suitable Lyapunov functions and designing delay-dependent feedback controllers, we derive several novel synchronization criteria, which guarantee the considered hybrid coupled networks can achieve fixed-time synchronization. Two numerical examples are given to show the effectiveness of our results.

1. Introduction

Over the past decades, it has been shown that complex networks [1–6] widely exist in the real world, such as World Wide Web, ecological networks, neural networks, electrical power grids, and so on. As the major collective behavior, the synchronization of complex networks has been extensively addressed due to its important applications in secure communication [7], image processing [8], automatic control [9], etc.

Although there have been many results about the synchronization control [10–13] of complex networks, most of them focused on asymptotic synchronization [14–16] and exponential synchronization [17–19], both of which can be incorporated in the category of infinite-time synchronization. However, in some engineering fields, it is more valuable that the synchronization can be realized in finite time, which leads to the studies of finite-time synchronization [20–25]. Compared with infinite-time synchronization, finite-time synchronization intrinsically requires a faster convergence speed, what is more, the states of the drive system and the response system remain completely identical after some finite time, which is called the settling time.

Note that a critical issue of finite-time synchronization is that the settling time is dependent on the initial synchronization error of the considered systems. Therefore, different initial synchronization error corresponds to different settling time. However, the initial conditions of many practical systems are usually impossible to be estimated, which results in the inaccessibility of the settling time. To overcome this difficulty, Polyakov introduced the concept of fixed-time stability and obtained some important theorems of fixed-time stability [26]. Different from common finite-time synchronization, the settling time of fixed-time synchronization is bounded by a fixed constant, which is independent of the initial synchronization error and can be estimated in advance once the suitable controller has been chosen. Up to now, there are few published papers dealing with the fixed-time synchronization of complex networks [27–29]. In [27], the fixed-time synchronization of Cohen-Grossberg neural networks was studied. In [29], the fixed-time synchronization of memristor-based BAM neural networks was investigated.

Since time delay is ubiquitous in nature, it is more meaningful to study the synchronization control of complex networks with time delay. The hybrid coupled complex networks considered in this paper have both internal delay and coupling delay, each of which is time-varying delay. It is noticed that if there is a signal transmitted from node j to node i, in most literatures the delayed coupling term is given by $D(x_j(t-\tau(t))-x_i(t-\tau(t)))$ [30–32]. However, because the time delay only affects the variable that is being transmitted from one system to another system, it is more reasonable to deem $D(x_j(t-\tau(t))-x_i(t))$ as the delayed coupling term. The hybrid coupled complex networks with one single time-varying delay coupling were proposed in [14,33], and it is illustrated that this model is more consistent with the reality. Based on the assumption condition $0 \le \dot{\tau}_i(t) \le h < 1,\ i = 1, 2$ and the as-

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sumption that the inner coupling matrices D and D_τ were positive semi-definitive diagonal matrices, the authors of Ref. [34] studied the finite-time synchronization of complex networks with one single time-varying delay coupling.

As far as we know, the result on the fixed-time synchronization of hybrid coupled complex networks has not been reported in the literatures until now. Therefore, it is interesting to fill this gap. In this paper, we propose a new complex network model with one single time-varying delay coupling, which is more general than that proposed in Ref. [34]. By constructing appropriate Lyapunov functions and designing delay-dependent feedback controllers, several new and effective criteria are derived to ensure the synchronization of the considered hybrid coupled networks can be realized in fixed time and in finite time, and we remove some unnecessary assumption conditions. Meanwhile, we also prove that the considered hybrid coupled networks can synchronize to its synchronization manifold in fixed time and in finite time. Two numerical examples are given to verify the effectiveness of our theoretical results.

The rest of this paper is organized as follows. In Section 2, some preliminaries are described. The main results are given in Section 3. In Section 4, two numerical examples are provided to illustrate the effectiveness of the obtained results. This paper is concluded in Section 5.

Notations. R^n denotes the n-dimensional Euclidean space, $R^{n\times n}$ is the set of $n\times n$ real matrices. The Euclidean norm is denoted as $\|\cdot\|$, accordingly, for vector $x\in R^n$, $\|x\|=\sqrt{x^Tx}$, where T denotes transposition; for matrix $A\in R^{n\times n}$, $\|A\|=\sqrt{\lambda_{\max}(A^TA)}$. For vector $e_i(t)=(e_{i1}(t),e_{i2}(t),\ldots,e_{in}(t))^T\in R^n$, $\|e_i(t)\|_1=\sum_{j=1}^n\left|e_{ij}(t)\right|$, $sign(e_i(t))=(sign(e_{i1}(t)),sign(e_{i2}(t)),\ldots,sign(e_{in}(t)))^T$, $sign^T(e_i(t))=(sign(e_{i1}(t)),sign(e_{i2}(t)),\ldots,sign(e_{in}(t)))$, $i=1,2,\ldots,N$.

2. Preliminaries

In this paper, we propose a new delayed hybrid coupled network consisting of N nodes, each node of which is an n-dimensional dynamical system

$$\dot{x}_i(t)=f(x_i(t))+g(x_i(t-\tau_1(t)))+\sum_{j=1,j\neq i}^{N}G_{ij}D(x_j(t)-x_i(t))$$
$$+\sum_{j=1,j\neq i}^{N}G_{ij}D_\tau(x_j(t-\tau_2(t))-x_i(t)),\ i=1,2,\ldots,N,\quad(1)$$

where $x_i(t)=(x_{i1}(t),x_{i2}(t),\ldots,x_{in}(t))^T\in R^n$ is the state variable of the ith node; f, g: $R^n\to R^n$ are continuous differentiable vector functions; $\tau_1(t)$ and $\tau_2(t)$ are the internal delay and the coupling delay respectively, and τ is the upper bound of $\tau_1(t)$ and $\tau_2(t)$; D and D_τ are the inner coupling matrices between node i and node j at time t and $t-\tau_2(t)$, respectively; $G=(G_{ij})_{N\times N}$ is the configuration matrix that satisfies the following conditions:

$$G_{ij}=G_{ji}\geq 0,\ i\neq j,\ G_{ii}=-\sum_{j=1,j\neq i}^{N}G_{ij},\quad(2)$$

where $G_{ij}>0$ if there exists a connection between node i and node j, and $G_{ij}=0$ otherwise. The initial condition of system (1) is given by $x_i(t)=\phi_i(t)\in C([-\tau,0],R^n)$, where $C([-\tau,0],R^n)$ denotes the Banach space of continuous functions mapping $[-\tau,0]$ into R^n.

We refer to system (1) as the drive system, then the corresponding response system can be written as follows:

$$\dot{y}_i(t)=f(y_i(t))+g(y_i(t-\tau_1(t)))+\sum_{j=1,j\neq i}^{N}G_{ij}D(y_j(t)-y_i(t))$$
$$+\sum_{j=1,j\neq i}^{N}G_{ij}D_\tau(y_j(t-\tau_2(t))-y_i(t))+u_i(t),$$
$$i=1,2,\ldots,N,\quad(3)$$

where $y_i(t)=(y_{i1}(t),y_{i2}(t),\ldots,y_{in}(t))^T\in R^n$ denotes the response state variable associated with the ith node, $u_i(t),i=1,2,\ldots,N$, are the appropriate controllers. The initial condition of system (3) is given by $y_i(t)=\varphi_i(t)\in C([-\tau,0],R^n)$.

Based on condition (2), we rewrite systems (1) and (3) as follows:

$$\dot{x}_i(t)=f(x_i(t))+g(x_i(t-\tau_1(t)))+\sum_{j=1}^{N}G_{ij}Dx_j(t)$$
$$+\sum_{j=1}^{N}G_{ij}D_\tau(x_j(t-\tau_2(t)))$$
$$-G_{ii}D_\tau(x_i(t-\tau_2(t))-x_i(t)),\ i=1,2,\ldots,N,\quad(4)$$

$$\dot{y}_i(t)=f(y_i(t))+g(y_i(t-\tau_1(t)))+\sum_{j=1}^{N}G_{ij}Dy_j(t)$$
$$+\sum_{j=1}^{N}G_{ij}D_\tau(y_j(t-\tau_2(t)))$$
$$-G_{ii}D_\tau(y_i(t-\tau_2(t))-y_i(t))+u_i(t),\ i=1,2,\ldots,N.\quad(5)$$

The synchronization errors are defined as $e_i(t)=y_i(t)-x_i(t)$, $i=1,2,\ldots,N$, then we can derive the following error system:

$$\dot{e}_i(t)=f(y_i(t))-f(x_i(t))+g(y_i(t-\tau_1(t)))$$
$$-g(x_i(t-\tau_1(t)))+\sum_{j=1}^{N}G_{ij}De_j(t)$$
$$+\sum_{j=1}^{N}G_{ij}D_\tau(e_j(t-\tau_2(t)))-G_{ii}D_\tau(e_i(t-\tau_2(t))$$
$$-e_i(t))+u_i(t),\ i=1,2,\ldots,N.\quad(6)$$

Throughout this paper, the following assumptions [35] are needed.

A1. For $\forall x,\ y\in R^n$, there exists a non-negative constant α such that

$$\|f(x)-f(y)\|\leq\alpha\|x-y\|.$$

A2. For $\forall x,\ y\in R^n$, there exists a non-negative constant β such that

$$\|g(x)-g(y)\|\leq\beta\|x-y\|.$$

Lemma 1 ([36]). *Let $x_1,x_2,\ldots,x_N\geq 0$, $p>1$, $0<q\leq 1$, then the following two inequalities hold:*

$$\sum_{i=1}^{N}x_i^p\geq N^{1-p}\left(\sum_{i=1}^{N}x_i\right)^p,\quad \sum_{i=1}^{N}x_i^q\geq\left(\sum_{i=1}^{N}x_i\right)^q.$$

Lemma 2 ([37]). *(Chain Rule) If $V(x)$: $R^n\to R$ is C - regular and $x(t)$ is absolutely continuous on any compact subinterval of $[0,+\infty)$, then $x(t)$ and $V(x(t)):[0,+\infty)\to R$ are differentiable for a.a.t $\in[0,+\infty)$, and*

$$\frac{d}{dt}V(x(t))=\upsilon(t)\dot{x}(t),\ \forall\upsilon(t)\in\partial V(x(t)),\quad(7)$$

where $\partial V(x(t))$ is the Clarke generalized gradient of V at $x(t)$.