Chaos, Solitons and Fractals 108 (2018) 71–76

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Fractal phase space and fractal entropy of instantaneous cardiac rhythm

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ARTICLE INFO

Article history:
Received 23 July 2017
Revised 19 January 2018
Accepted 21 January 2018

Keywords:
Instantaneous cardiac rhythm
Phase space
Extended phase space
Fractals
Phase space volume
Entropy

ABSTRACT

The paper is devoted to the study of cardiac rhythm variability (CRV) using the phase and extended phase spaces of instantaneous cardiac rhythm (ICR) obtained from the Holter monitoring (HM) data. In order to construct these spaces, a software package is developed and implemented. With specific references, the fractality of the ICR phase space is demonstrated. The fractal phase space volume and fractal entropy definitions of ICR are given. The paper justifies their availability for CRV quantitative assessment.

1. Introduction

Today the basic approach to the assessment and prediction of fatal cardiovascular complication development risk is the analysis of cardiac rhythm variability (CRV). The Consensus of the Working Group on CRV study of the European Society of Cardiology and the North American Society of Pacing and Electrophysiology states the regulations for the clinical relevance of CRV analysis in patients suffering from ischemic heart disease. According to this document, decreased CRV is an independent predictor of increased risk of formation of life threatening arrhythmias and sudden cardiac death (SCD) from myocardial infarction [1]. The latter is a major problem both medically and sociologically, since its incidence is very high. Every year about 400,000 Americans die suddenly, among them 250,000 die from phenomena classified as SCD.

The powerful CRV research methods include mathematical ones. They allow one to identify and use hidden and important regularities observed in the course of CRV studies. One of the first papers in this area was written by R. M. Bayevsky et al. [2]. At present a prominent place in this area is occupied by the papers dedicated to CRV study based on the fractal and chaos theory [3–6]. Our paper considers the further development of this research trend.

One of the upcoming CRV study trends is the instantaneous cardiac rhythm (ICR) analysis [7,8] in different time intervals: short (lasting for 1–2 min), medium (within about 1 h) and long (lasting for a day and longer). In order to perform this analysis, new high performance techniques for Holter monitoring (HM) big data [7,8] analysis are required.

For this purpose we will construct the phase and the extended phase spaces of ICR. Basing on the fractality of the PS of ICR and allowing for the fractal properties, the values of its fractal phase space volume Γ and the corresponding value of fractal entropy S will be introduced. At that, we will demonstrate that the values of fractal dimension D, fractal phase space volume Γ and fractal entropy S of ICR adequately reflect the CRV.

Let i be an RR-interval number, $i = 1, 2, 3, \dots N$. Upon the expiration of 24 h of HM, the N value will essentially exceed $100,000$. The time points t_i will correspond to ECG R-wave peaks. Then $T_{RR_i} = t_{i+1} - t_i$, and the ICR values y_i in time points t_i will be as follows:

$$y_i = \frac{60}{T_{RR_i}} = \frac{60}{t_{i+1} - t_i}. \tag{1}$$

In Eq. (1), the time points t_i are measured in seconds, and the ICR values y_i in min^{-1}.

In the time interval $t_i \le t \le t_{i+1}$, the equation of ICR curve can be presented as follows:

$$y(t) = y_i + \frac{y_{i+1} - y_i}{t_{i+1} - t_i}(t - t_i). \tag{2}$$

Together with $y(t)$, let us introduce the ICR change rate $v(t)$. From this point on, the numerical values of functions $y(t)$ and $v(t)$ will be represented in min^{-1} and sec^{-1} respectively. At the point t_i, we define the values v_i of the function $v(t)$ as a difference deriva-

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https://doi.org/10.1016/j.chaos.2018.01.030

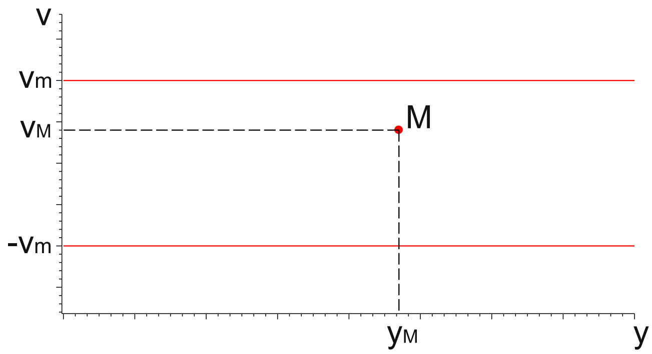

Fig. 1. A phase space of ICR.

tive of the function $y(t)$:

$$v_i = \frac{y_{i+1} - y_i}{t_{i+1} - t_i}. \tag{3}$$

Then in the interval $t_i \le t \le t_{i+1}$, the function $v(t)$ is given by

$$v(t) = v_i + \frac{v_{i+1} - v_i}{t_{i+1} - t_i}(t - t_i). \tag{4}$$

The functions $y(t)$ and $v(t)$ contain the exhaustive information on the ICR pattern in the time interval of interest, and in the future we will use them for constructing the phase space of ICR.

2. The ICR phase space

From this point on, we will use the concept of the ICR phase space to the same significance as in theory of dynamic systems. The phase space is a space of all possible system states to each of which a phase space point, or a phase point corresponds. The phase point coordinates give a quantitative description of the basic system parameters, in this case the ICR at the given time moment. The dynamic system evolution will be described by the motion of the phase point along the phase trajectory. In the Newtonian mechanics, in the theory of differential equations, and in the theory of dynamic systems, the phase trajectories are smooth manifolds. In the case of ICR, they do not have this property. By specific examples we will show that the phase trajectories of ICR are reminiscent of Brownian motion trajectories and thus are fractal curves.

We will call the set of points M in $\mathbb{R}^2$ with orthogonal coordinates $y(t)$ and $v(t)$ a phase space (PS) of ICR. The points of this space adequately reflect all characteristics of ICR in the time interval of interest.

Let us illustrate this definition in Fig. 1.

The line $-v_m \le v \le v_m$ separates the phase space of ICR into two regions: the region of normal ICR, where $|v| \le v_m$, and the region of ICR jumps (catastrophes), where $|v| > v_m$.

Let us evaluate v_m according to the ratio:

$$|v_i| < \frac{1}{60}\left|\frac{T_{RR_{i+1}}}{T_{RR_i}} - 1\right|\left(1 - \left|\frac{T_{RR_{i+1}}}{T_{RR_i}} - 1\right|\right)^{-1} y_i^2$$

In cardiointervalography, for a patient being in normal condition $max|T_{RR_{i+1}}/T_{RR_i} - 1| < 0.1$, $y_i < 90$. Then we the following estimate takes place: $v_m = max|v_i| = 15$.

If $v_m \sim 1$, then $y(t)$ can be described in terms of a piecewise linear trend and small deviations from it. When v_m increases by an order of magnitude, i.e. to the value $v_m \sim 10$ and greater, such separation becomes impossible. A brand new ICR state arises which is called a region of ICR jumps (catastrophes) [9].

In Ref [9], it was demonstrated that the catastrophes in multifractal dynamic systems could be more adequately described within the framework of the multifractal dynamics (MFD) model.

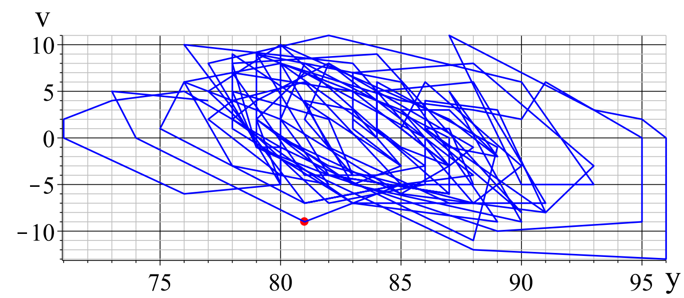

Fig. 2. The phase space of ICR for the first patient.

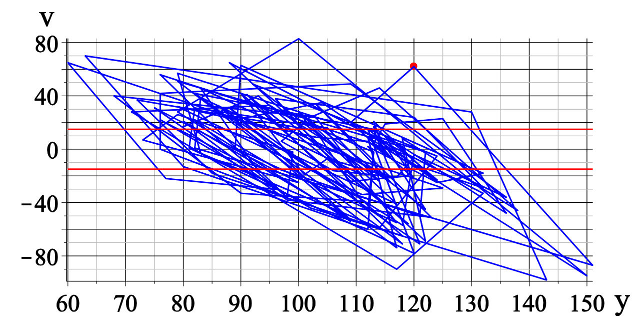

Fig. 3. The ICR phase space for the second patient.

HM data recording and analysis were performed using complex "Kardiotekhnika-04" (from INKART, St. Petersburg) and the program system developed by the corporate authors. As the experience shows, the ICR states on the phase diagram visually differ if $N \le 150$, which is equivalent of a time interval of the order of 1–2 min.

Consider an example of constructing the PS of ICR for one of the patients of Tver Regional Cardiology Health Center for the HM time interval of 1.5 min. It is demonstrated in Fig. 2.

In this figure, the heavy point reflects the phase point position at an arbitrary point of observation time. Directly in Fig. 2, it is difficult to observe some phase point dynamics. As a consequence we wrote down the real-time ICR phase point motion animation application. The visual observation of the phase point motion pattern allows performing detailed analysis of ICR dynamics and, consequently, the patient cardiovascular system condition. As seen from Fig. 2, the phase point trajectory has a Brownian pattern in a quality manner and would be a fractal curve. Hereafter we will confirm this property by symbolic-numeric calculations.

For comparison, we also construct the PS of ICR for the second patient of Tver Regional Cardiology Health Center (Fig. 3) according to the HM data for the similar time interval of 1.5 min, as in the case of the first patient.

By comparison of the phase trajectory patterns in Figs. 2 and 3, we can see that in the case of the second patient, the phase trajectory of ICR is more chaotic, i.e., the fractal dimension D is expected to be essentially higher.

As seen from Fig. 3, during the most of the time the phase point is located in the ICR jump region $|v| > v_m$.

3. The extended ICR phase space

Together with the PS of ICR, let us introduce the extended phase space (EPS) of the ICR. For this purpose, let us introduce the values of n_i, the number of repeatability of points with coordinates y_i and v_i. We will construct the function $n(t)$ similarly to the functions $y(t)$ and $v(t)$. In the time interval $t_i \le t \le t_{i+1}$ let us assume