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Quantum chaos analysis for characterizing a photonic resonator lattice

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ABSTRACT

Tailoring the propagation of light in an arbitrarily manner has motivated a great of interest on nanophotonics. As a new mechanism for this purpose, the generation of an effective magnetic field leading to a Lorentz force for photons is recently proposed in a photonic resonator lattice. Here, we consider a photonic resonator lattice with a harmonically modulated phase and with an interface splitting the lattice into two magnetically different regions. Considering this lattice, we try to explore the impact of phase and the location of interface on the localization of Hamiltonian eigenstates by applying level spacing distribution as a cornerstone of random matrix theory. The obtained results show that while the location of interface has no effect on the appearance of localized states in weak phases, in strong phases it is found a threshold value for location of interface above which all eigenstates are delocalized. As a result, level spacing distribution and so random matrix theory is capable of characterizing the behavior of a photon in regions with different magnetic properties.

1. Introduction

Thanks to fundamental breakthroughs and disruptive applications, optics and the science of light has been a focus of great interest in past decades [1]. Discovering that Anderson localization is a wave phenomenon paved the way towards the application of localization and control concepts to electromagnetic [2,3] and especially to light waves [4–6]. Regulating the propagation of light is critical for potential applications in on-chip communications, information processing, biotechnology, medicine and the modern-day telecommunications industry [7–9]. In this regard, the past decade has witnessed considerable progress in nanophotonics, as the main optics-related discipline to study the light-matter interaction [10], an in developing synthetic materials and nano-scale photonic devices capable of guiding light in a controllable manner [11]. Despite the difficulty of tailoring the propagation of light with light, due to vanishing photon-photon interaction cross sections [12], various approaches have been proposed to this end with the help of nonlinear optical effects [13,14], optical resonators [15], exciton polaritons [16,17], or with the optical response of a two-level system in a single fluorescent molecule [18].

Regarding the electron transport, externally-applied electric and magnetic fields are of great importance for both the classical and quantum regimes [19], imposing a Lorentz force on the classical level [20] and an integer and fractional quantum Hall effects [21,22] on the quantum level. Despite electrons, photons do not carry electrical charge, and therefore there is no natural gauge potential which couples to a photon [23]. Realizing similar mechanisms for guiding light in a controllable way is a rapidly rising field of research, addressing the establishment of photonic phenomena due to the effective magnetic field (EMF) similar to charged particles subjected a real magnetic field [24]. On this matter, different approaches covering harmonically modulating the refractive index of the photonic crystal [23], and harmonically modulating coupling constants between the resonators in two [25,26] and three dimensions [27] were proposed in recent years. Accordingly, Aharonov-Bohm [23,28], and the photonic de Haas-van Alphen [19] effects caused by an effective gauge field for photons were numerically and/or experimentally investigated.

Thus far, less attention has been paid on the impact of the EMF on the localization and diffraction of photons in photonic devices. Regarding the importance of localization, the present study has aimed at tailoring and controlling the flow of light by Hamiltonian engineering. For this purpose, besides investigating the dynamics of the participation ratio we also turn our attention to the spectral fluctuations addressing stability/instability of optical modes [29] and compare obtained results with the predictions of random matrix theory. Random matrix theory, firstly established by Wigner [30], can be applied to unveil the underlying physics of complex systems and gives a transparent characterization based on the fluctuation measures of the states.

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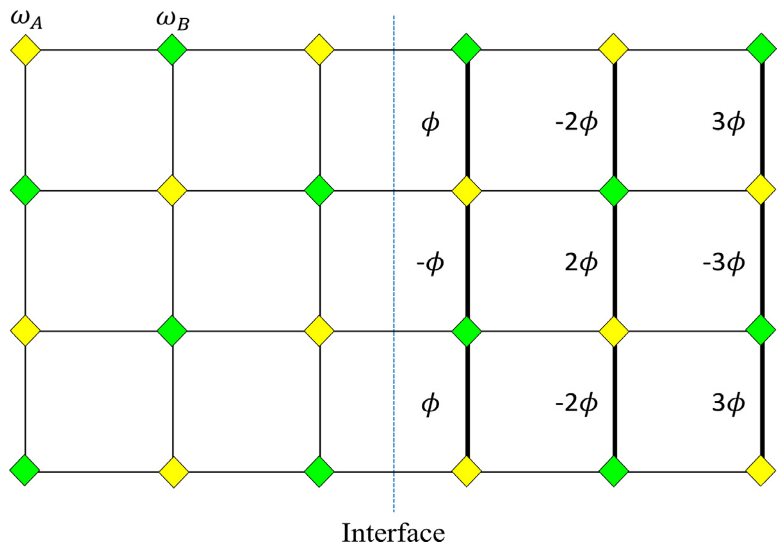

Fig. 1. The schematic picture of a dynamically modulated photonic resonator lattice with an interface separating left region (without EMF) from right region (exhibiting an EMF). The square sublattice of resonators with frequency $\omega_A(\omega_B)$ is denoted by yellow (green) diamond. The light black lines represent nearest-neighbor couplings with zero phase of the coupling. The Bold black vertical lines hold for non-zero phase of the coupling proportional to column index with flipping feature between two neighboring bonds within the same column. (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

The remaining of the manuscript is assembled as follows. Section 2 describes the studied Hamiltonian. Section 3 is devoted to the characterization of the localization of eigenstates by focus on the dynamics of the participation ratio. Section 4 contains the fluctuation statistics of levels by a direct attention on level spacing distribution. Finally, the obtained observations are concluded in Section 5.

2. Tight-binding model

The tight-binding model describing the creation of an EMF for photons in a two-dimensional photonic resonator lattice [25] with 24 resonators is depicted in Fig. 1. Each square unit cell of the lattice contains two resonators A (symbolized by yellow diamonds) and B (symbolized with green diamonds) with different frequencies ω_A and ω_B, respectively. Notably that only nearest-neighbor coupling with a harmonically modulated feature between the two sublattices are considered. The Hamiltonian of the described resonator lattice is read as

$$H = \omega_A \sum_i a_i^\dagger a_i + \ \omega_B \sum_j b_j^\dagger b_j + \sum_{<ij>} V\ cos(\Omega t + \phi_{ij})(a_i^\dagger b_j + b_j^\dagger a_i), \tag{1}$$

where $a_i^\dagger(a_i)$ and $b_j^\dagger(b_j)$ create (annihilate) a photon in the ith and jth resonators from the A and B sublattices, respectively, the $<ij>$ represent that i and j are nearest neighboring sites. $V = 2$ [25] holds for the coupling strength, Ω is the modulation frequency. And, ϕ_{ij} stands for the phase of the modulation between adjacent resonators at sites i and j. Here we perform our study on a square lattice with 40 resonators in each direction.

In the rotating wave approximation $V \ll \Omega$ provided that the modulation is on resonance $\Omega = |\omega_A - \omega_B| = 100$ [25], the counter rotating term of Hamiltonian can be discarded [19]. In this approximation we find

$$H = \sum_{<ij>} \frac{V}{2}(e^{-i\phi_{ij}} c_i^\dagger c_j + e^{i\phi_{ij}} c_j^\dagger c_i), \tag{2}$$

where $c_{i(j)} = e^{i\omega_{A(B)}t} a_i(b_j)$. Hamiltonian (Eq. (2)) is similar to the Hamiltonian of a charged particle on a lattice imposed to a magnetic field with the following association

$$\int_i^j \mathbf{A}_{eff} d\mathbf{l} = \phi_{ij}. \tag{3}$$

It is found that a uniform EMF,

$$B_{eff} = \frac{1}{d^2} \oint_{Plaquette} \mathbf{A}_{eff} d\mathbf{l} = \frac{\phi}{d^2}, \tag{4}$$

is imposed on photons by assigning the special distribution of the modulation phase as shown in Fig. 1, where it is supposed that all bonds along the horizontal and vertical directions, denoted by light lines, have zero phases. Bold lines along the vertical direction represent location-dependent modulation phases [25]. d is the spatial distant between any two adjacent resonators.

3. Localization of eigenstates

As respects the manipulation of photon propagation, diffraction of light beams brings an additional obstacle [31]. In this regard, the localization of light draws considerable attention [32]. While Anderson explored the localization theory in disordered and periodic scattering lattices [33], the past two decades have witnessed growing attention on the observation of localization in lattices without any defects [34–38].

One of the striking features of the states under sufficiently strong magnetic fields in disordered dense atomic systems is their localization [39,40]. In addition to the magnetic field, we try here to reveal the impact of location of interface (LI) on the localization properties of eigenstates of the defined system (see Fig. 1). LI is the exact location of the Interference that separation the phases, based on the site numbers. To characterize a state $|\Psi^{(n)}> = \sum_i C_i^{(n)} |\psi_i>$ pertinent to the nth eigenvalue in quantum regime, it is common to chose the participation ratio ($PR^{(n)} = \frac{1}{\sum_i |C_i^{(n)}|^4}$) as a well-known measure of localization. $PR^{(n)}$, reflecting some valuable information about the spatial distribution of the eigenstate $|\Psi^{(n)}>$ expanded in the basis vectors $|\psi_i>$ of the corresponding Hilbert space, measures how extended a given state is in the defined basis. Thus for a completely localized state, i.e., a state similar to one of the basis vectors, PR equals 1. In contrast, for a fully extended state, i.e., a state with uniform superposition of all the basis vectors, PR equals the size of the corresponding basis.