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Complex neutrosophic concept lattice and its applications to air quality analysis

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ABSTRACT

In the current year, the precise measurement of uncertainty and fluctuation exists in a complex fuzzy attributes is addressed as computationally and mathematically expensive tasks with regard to its graphical analytics. To deal with this problem the calculus of complex neutrosophic sets are recently introduced to characterize the uncertainty and its changes based on its truth, indeterminacy, and falsity membership–value, independently. This given a way to represent the given data sets in form of complex neutrosophic matrix for further analysis towards knowledge processing tasks. In this process, a major problem arises when an expert wants to find some of the interesting patterns in the given complex neutrosophic data sets to solve the particular problem. To resolve this issue, the current paper proposes a method for step by step demonstration to investigate the complex neutrosophic concepts and their graphical structure visualization based on their Lower Neighbors. One of the suitable examples of the proposed method is also given for precise measurement of uncertainty exists in Air Quality Index (AQI) and its pattern at given phase of time.

1. Introduction

Recently, the calculus of complex vague set concept lattice [25] and its properties [26–28,34,35] is introduced for measuring the changes in uncertainty using amplitude and phase term of a complex set. It has given a new mathematical way to characterize the uncertainty and vagueness in attributes more understandable manner when compared to approaches available in unipolar fuzzy space [11,16,23,24]. The reason is that the calculus of complex set [12,13] and concept lattice theory [15,24] provides a well-established mathematical framework to measure the human cognitive thought [14]. To measure the fluctuation in uncertainty exists in fuzzy attributes the calculus of complex fuzzy sets [12,13] become more helpful in its precise representation using amplitude and phase term in bipolar [22–25] or three–way space [26,27] for multi–decision process [28]. In this process, an important problem was addressed while handling the three–way fuzzy attributes [26–28] that how to measure their changes at given phase of time. To achieve this goal, properties of complex neutrosophic sets [3–5] and its graphs [10] are introduced for handling multi–decision attributes [9,25,28,33]. This extensive version of complex fuzzy set [1,2,29,30] and its properties in the neutrosophic or three–way polar space [31–33] given a new orientation to analyze the data sets based on applied abstract algebra [1,2,15,21,37–39]. Towards this extension recently, Singh [26–28] introduces properties of neutrosophic [26,27] and complex vague set [25] based concept lattice for precise approximation of computational linguistics exists in three–way decision space [47,48]. In this process a problem is addressed while handling the changes in three–way fuzzy attributes based on its truth, indeterminacy and falsity membership–values [3–5,10]. One of the suitable example is 22 °C temperature used to consider as a cool in summer seasons, warm in winter season whereas fair (or uncertain) in spring season. This interpretation of human cognitive thought used to exists in several real life examples from morning to evening while taking veg, non-veg or indeterminant spices. The precise representation of these types of attributes using a mathematical model is rigorous tasks for the research communities. This problem is dovetail which affect the human life directly in form of Air Quality Index (AQI)[1] or Bushfire.[2] All of these cases characterizing the uncertainty and fluctuations based on its acceptation, rejection and uncertain part is major concern. Second problem arises with their mathematical representation and graphical analytics for further analysis. Hence the current paper focuses on solving these issues of complex fuzzy attributes. The motivation is to provide a mathematical model for easier of understanding the information contained in complex neutrosophic data sets

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[1] https://en.wikipedia.org/wiki/Air_quality_index.
[2] https://en.wikipedia.org/wiki/Bushfires_in_Australia.

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0960-0779/

Table 1
Some necessary conditions for the uses of complex neutrosophic set.

	Complex fuzzy set	Complex vague set	Complex neutrosophic set
Domain	Universe of Discourse	Universe of Discourse	Universe of Discourse
Co–domain	Unipolar–value in unit circle [0, 1]	Bipolar–valued in unit in circle [0, 1]	Three–valued in unit circle $[0, 1]^3$
Truth membership	Yes in [0, 1]	Yes in $[0, 1]^2$	Yes in $[0, 1]^3$
False membership	No	Yes in [0, 1]	Yes in $[0, 1]^3$
Indeterminacy membership	No	1–True –false	Yes in $[0, 1]^3$
Amplitude term	Yes in [0, 1]	Yes in $[0, 1]^2$	Yes in $[0, 1]^3$
Phase term measurement	Yes $[0, 2\pi]$	Yes $[0, 2\pi]$	Yes in $[0, 2\pi]$
Uncertainty measurement	Yes in [0, 1]	Yes in $[0, 1]^2$	Yes in $[0, 1]^3$
Fluctuation measurement	Yes	Yes	Yes
Graph	Yes	Yes	Yes

based on its maximal acceptation, minimal rejection or uncertain regions, independently. To achieve this goal, the current paper focuses on depth analysis of complex neutrosophic context and its graphical structure visualization based on applied abstract algebra.

Recently, some of the researchers started analyzing the indeterminacy based on their partial ordering [17–20] graphical visualization [26–28] to approximate it more prominently via three–way decision space [40–42]. All of these approaches fail in precise measurement of periodic changes in three–way or neutrosophic fuzzy attributes. One of the suitable example is Air Quality Index (AQI) of any country changes at each interval of time. In this case, measuring the AQI based on its acceptation, rejection or uncertain regions is a computationally expensive tasks for the researchers. The reason is that the values of AQI used to change several times in a day due to change in level of $PM_{2.5}$, PM_{10}, NO_2 and other parameters. In this case precise representation of uncertainty and its changes based on its acceptation, rejection and uncertain regions at given phase of time is mathematically expensive tasks. To conquer this problem recently, some of the researchers tried to represent them using the calculus of complex neutrosophic sets [3–5] and its graph theory [10] for multi–decision process [9,18,28,36] at δ–granulation [43–48]. However, none of the available approaches described any ways to find some of the useful pattern exists in the complex neutrosophic contexts for knowledge processing tasks. Due to which, the current paper focuses on introducing a method for finding some of the interesting pattern in complex neutrosophic contexts based on applied lattice theory. The reason is it provides a more descriptive measurement of uncertainty and its changes in the complex fuzzy attributes based on its truth, indeterminacy and uncertain regions, independently when compared to other extensions of neutrosophic sets as shown in Table 1. To acquire this advantages the calculus of applied lattice theory [11,15,16,38,39] and its extensive properties [22–28] is utilized in this paper for generating the complex neutrosophic concepts and its hierarchical order visualization in the concept lattice using their Lower Neighbors. The reason to utilize the Lower Neighbor method is that it provides an easier way to investigate the concepts within limited time complexity when compared to other approaches [6–8]. In this way, the proposed method provides a basis of an algorithm for compressed graphical visualization of complex neutrosophic context in the concept lattice. The motivation is to provide a mathematical model to analyze the complex neutrosophic data sets more precisely when compared to its numerical representation. The objective is to extract some of the useful pattern in the given complex neutrosophic context for multi–decision process as shown in Fig. 1. It can be considered as one of the significant outputs of the proposed method in the field of complex data set analysis.

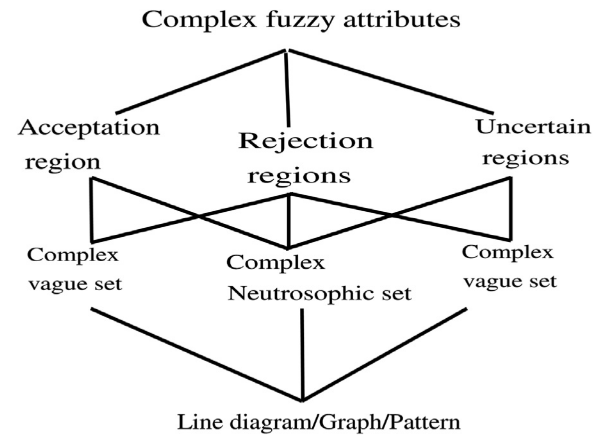

Fig. 1. The motivation for introducing the complex neutrosophic concept lattice.

Remaining part of the paper is organized as follows: Section 2 provides some basic preliminaries about complex neutrosophic sets. Section 3 provides a method for generating the complex neutrosophic concepts using their Lower Neighbor. Section 4 provides illustration of the proposed method with an example. Section 5 contains discussions followed by conclusions, and references.

2. Complex neutrosophic context and its graphical visualization

Recently, it seems that. handling complex neutrosophic data set like measuring the quality of AQI is mathematically rigorous tasks. To deal with these types of complex or seasonal data sets one solution is to represent them matrix format and try to visualize them in the graph. The current section contains some useful definitions to achieve this goal as given below:

Definition 1 (Complex fuzzy set [21,29,30])**.** A complex fuzzy set Z can be defined over a universe of discourse U having a single fuzzy membership–value at given phase of time. The complex–valued grade of membership of an element $z \in U$ can be characterized by $\mu_Z(z)$. The membership–values that $\mu_Z(z)$ may receive all values within the unit circle of a defined complex plane in the form $\mu_Z(z) = r_z(x)e^{iw_z(x)}$, where $i = \sqrt{-1}$, both $r_Z(z)$ and $w_Z(z)$ are real–valued and $r_Z(z) \in [0, 1]$. The complex fuzzy set Z may be represented as the set of ordered pairs:

$$Z = \{(z, \mu_Z(z)) : z \in U\} = \left\{(z, r_Z(z)e^{iw_Z(z)}) : z \in U\right\}$$

The union, intersection and other operator among complex fuzzy set can be studied in [1–3] with an illustrative example for better understanding.

Example 1. Let us suppose, an expert wants to measure the level of AQI index of the given geographical regions (i.e. object–x_1) based on its saturation value of PM_{10} (i.e. attribute y_1). The user collected the data and saw that the saturation value of PM_{10} changes 50 percent in six to seven months. This complex fuzzy attributes can be written using the properties of complex fuzzy set as follows: $0.5e^{i1.2\pi}$. In case the user want to represent the indeterminacy and falsity regions then properties of neutrosophic set can be useful.

Definition 2 (Neutrosophic set [32])**.** It provides a way to characterize the uncertainty and vagueness in attributes $y \in Y$ based on truth–membership function $T_N(y)$, a indeterminacy–membership