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Synergy punishment promotes cooperation in spatial public good game

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ABSTRACT

In previous work, punishment has been proved to an effective way to promote cooperation, and punish is introduced as a strategy just like cooperate or defect. In this paper, however, we introduce a new synergy punishment mechanism into a spatial public good game which is different from the previous works, every cooperator has a probability to be a punisher, additionally if punishment is carried out by several punishers, the cost of punishment will be reduced by synergistic effect. The simulation results show that punishment can promote cooperation and the synergistic effect has an obvious influence on cooperation for a small punishment fine but has little influence for a big punishment fine.

1. Introduction

The emergence of cooperation is a ubiquitous phenomenon in biological and social systems [1–5]. It remains to be an interesting problem to study and search for mechanisms that can generate and maintain the cooperation among the egotism individuals. Evolution game theory provides a fruitful mathematical framework to model and elucidate the evolution of cooperation among selfish individuals [6–8]. For example, prisoner dilemma game (PDG), the snowdrift game (SDG) and the stag-hunt game (SHG) have been used to study the cooperation between pairwise interactions, and attracted a lot of attention. However, some social dilemma involves a large group of interactional individuals. In that case, the public good game (PGG) provides a powerful framework to address the issue [6].

In recent years, aimed at solving the social dilemma, a great number of approaches have been proposed. Nowak reviewed five rules for the promotion of cooperation named kin selection, direct reciprocity, indirect reciprocity, network reciprocity, and group selection in 2006 [9,10]. In particular, network reciprocity, is a well-known dynamical rule that fosters the prevalence of cooperation, has inspired many works to investigate the evolution of cooperation on networks including regular lattice [11–22], small-world network [23], scale-free network [24–28], interdependent network [29].

Punishment has been proved to be an effective mechanism to sustain cooperation among selfish individuals [30–34]. A punisher is a player who bears a cost in order to punish an anti-social defector and therefore justice is done. However, the Achilles' heel of punishment is the fact that it is costly, which is the main obstacle for people to stand out and uphold justice [35–45]. In reality, the price can be reduced by joining our hands to defeat the defectors. This assumption has a wide background in the human society and animal world. In East Africa Savannah, for example, although a jackal shrinks when it confronts a lion, but on the contrary, a lion shrinks when it confronts a herd of jackals. As a result, we can have the conclusion that power can be multiplied be joining hands. As an old saying goes, many hands make light work. If people combine to carry out the punishment, the price of punishment that each punisher bears will be reduced sharply. Inspired by that, we introduce a synergistic factor γ to reduce the price which punishers must bear.

Where there is oppression there is resistance. People will stand out if the circumstance is bad enough to expand the enduring limit. So, in this paper, a cooperator has more probability to become a punisher as the condition of the group becomes worse that is to say more defector exist in the group.

2. Method

In our model, players are randomly located on a $L \times L$ periodical boundary condition square lattice. Every player occupies a lattice point and has four neighbors. Each player participates in five public good games described as follows.

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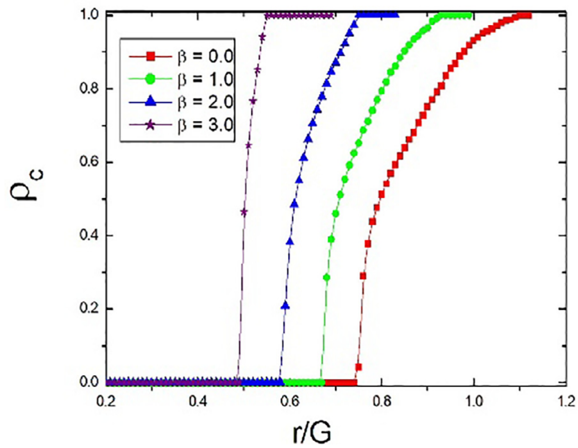

Fig. 1. The fraction of cooperator ρ_c in dependence on the normalized enhance factor r/G for different fine β. For each value of β, ρ_c increases as r/G increases. For $\beta = 0$, the condition goes back to the traditional public good game. As β increases, the cooperators emerge at a smaller value of r/G and ρ_c increases more sharply. There result is obtained by setting $\gamma = 1$.

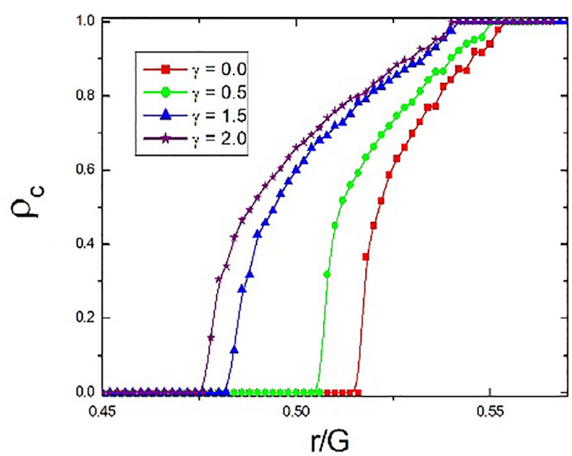

Fig. 2. The fraction of cooperator ρ_c in dependence on the normalized enhance factor r/G for different value of synergistic factor γ. For each value of γ, ρ_c increases as r/G increases. As γ increases, cooperator emerge at smaller enhance factor. And for a fixed enhance factor, ρ_c becomes larger as γ becomes larger. These result is obtained by setting $\beta = 3$.

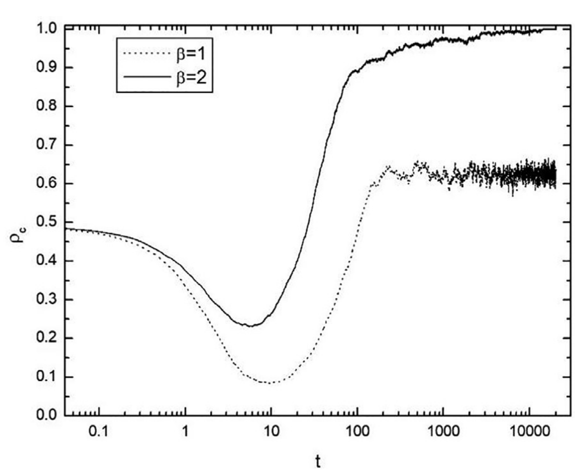

Fig. 3. The fraction of cooperator ρ_c in dependence on the time step t for different values of the punishment fine β. As time evolves, ρ_c decreases at first and then increases to a steady value. The result is obtained by setting enhance factor $r = 3.75$, and synergistic factor $\gamma = 1$.

In typical public good game, each player is surrounded by its $k = G - 1$ direct neighbors and is a member of $g = G$ different groups. Initially, each player on these two networks is designed either as a cooperator or defector with equal probability. Cooperators contribute 1 to the common pool and defectors contribute nothing in each group. The total contribution is subsequently multiplied by an enhancement factor r and then equally shared by G group members irrespective of their strategies. Thus, we can calculate the players' payoff P_x^g, which can be expressed as:

$$P_x = \sum_{x \in \Omega_x} P_x^g \qquad (1)$$

Where Ω_x denotes the community of neighbors of x and itself. The more players decide to cooperate, the more payoff of the whole group will be got. Whereas, for an egoistic individual choosing to defect is always better than to cooperate regardless of the group composition. The social dilemma occurs because the best strategy for a selfish individual and that for the group do not coincide.

After every PGG game, on one hand, every cooperator in a group will become a punisher with the probability $N_D/5$, where N_D denotes the number of the defector in the group. The more defectors, the more likely a cooperator choose to be a punisher. Once the cooperator chooses to punish, he will pay a price:

$$\frac{1}{N_p^{\gamma}} \times \frac{N_D * \beta}{N_P} \qquad (2)$$

where N_P denotes the number of the punisher in the group, γ is the synergistic factor varies from 0 to 2. It is obvious that if there is more than one punisher in a group, the price of punishment can be reduced by synergy effect. On the other hand, every defector in a group will be fined a punishment β if there is any punisher in the group.

After calculating the final payoff, each individual i randomly chooses a neighbor j, then update his strategy with the probability:

$$W_{(S_y \leftarrow S_x)} = \frac{1}{1 + \exp\left[(P_x - P_y)/K\right]} \qquad (3)$$

where K denotes the amplitude of noise or its inverse the so-called intensity of selection. In the $K \to 0$ limit, player y imitates the strategy of player x if and only if $P_x > P_y$. Conversely, in the $K \to 1$ limit, payoffs seize to matter and strategies change as toss coin. Following the previous studies, here we set $K = 0.5$ [45,46]. During one full Monte Carlo step (MCS) each player has a chance to adopt one of the neighboring strategies once on average.

The results of Monte Carlo simulations presented below were obtained on 100×100 lattices. The key quantity the fraction of cooperators ρ_c was determined within the last 5×10^3 full MCS over the total 2×10^4 steps. Moreover, since the popularity driven selection process may introduce additional disturbances, the final results were averaged over up to 100 independent realizations for each set of parameter values in order to assure suitable accuracy.

3. Result

We start by examining the influence of the punishment. Fig. 1. shows how ρ_c varies in dependence on the normalized enhance factor r/G for different values of β. For any given value of β, ρ_c increases from 0 to 1 as r/G increases. We can observe that, for $\beta = 0$ (namely, the classical version), that cooperators will mushroom when r/G is at around 0.75. When β increases to 1.0, which means the punishment is introduced, the cooperators emerge at smaller value of r/G and even expand more rapidly as r/G increases. The results suggest that the punishment mechanism can significantly sustain the emergence and evolution of cooperation. Fig. 2 shows how ρ_c varies in dependence on the enhance factor r