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Analytical solutions for conformable space-time fractional partial differential equations via fractional differential transform

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ABSTRACT

This paper introduces an efficient fractional differential transform that is called "conformable fractional partial differential transform (CFPDT)" and its properties for solving linear and nonlinear conformable space-time fractional partial differential equations (CSTFPDEs). Moreover, a CFPDT is more practical and helpful for solving abroad CSTFPDEs. Analytical solutions to linear Navier–Stokes equation and nonlinear gas dynamic equations in sense of conformable space-time fractional partial derivatives are successfully obtained to confirm the accuracy and efficiency of the proposed transform.

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1. Introduction

Fractional partial differential equations (FPDEs), as generalizations of classical partial differential equations, have been proposed and investigated in many research fields, such as fluid mechanics, mechanics of materials, biology, plasma physics, finance, chemistry, image processing and other areas of science and engineering, see [1–3] and some references cited therein. Many proposed definitions from a different point of view of applications have been introduced in the literature. Of recent, scientists began to see deficiencies in most of the fractional derivative definitions and why some real-life problems could not be captured. Moreover, there are certain functions which do not have Taylor power series representation or their Laplace transform cannot be calculated. Consequently, a rather simple and intriguing definition called "the conformable fractional derivative" was proposed by Khalil et al. in [4], which is depending just on the basic limit definition of the derivative. Through this a new definition, many most recent works have been done, see for example [5–11] and some references cited therein. The conformable fractional partial differential equations (CFPDEs) are simply PDEs in sense of conformable partial fractional derivatives. In a study of FPDEs or CFPDEs, one should note that finding an analytical or numerical solution is a challenging problem.

Since the exact solutions to CFPDEs are rarely available, so analytical and numerical methods are applicable. Therefore, accurate methods for finding the solutions of CFPDEs are yet under investigation. Several analytical and numerical methods for solving CFPDEs exist in the literature, for example: First integral method [12] where the method was combined with conformable fractional derivative for finding traveling wave solutions of nonlinear fractional partial differential equations, modified Kudryashov method [8] where the method was applied to solve the conformable time-fractional Klein–Gordon equations with quadratic and cubic nonlinearities, tanh method [13] and the method was applied to find analytic solutions of the conformable space-time fractional Kawahara equation and recently in [14] Melike Kaplan applied two reliable methods for solving a nonlinear conformable time-fractional equation.

The goal of this paper is to modify the theory of conformable fractional calculus by introducing an efficient reliable CFPDT in which linear and nonlinear CSTFPDEs can be solved easily. The rest of the paper is organized in as follows: In Section 2, we present basic definitions and theorems of fractional and conformable fractional derivatives which are needed in the sequel. In Section 3, we introduce a CFPDT for solving CFPDEs. Approximate analytical solutions for linear Navier–Stokes equation and nonlinear homogeneous, non-homogeneous gas dynamic equations in sense of conformable space-time fractional derivatives are obtained in Section 4.

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2. Preliminaries

There are various definitions and theorems of fractional integrals and derivatives. In this section, we give some definitions and theorems of the fractional and conformable fractional calculus theory. Some of these definitions and theorems are new in this paper, and some others can be found in [15–17] and some references cited therein.

2.1. Fractional partial derivatives

Definition 2.1. Let $\alpha \in \mathbb{R}$, $m-1 \le \alpha < m \in \mathbb{N}$, the Riemann–Liouville time fractional partial derivative of order α for the function $u(x, t)$ is defined as follows:

$$^{RL}\mathcal{D}_t^{\alpha} u(x,t) = \frac{\partial^m}{\partial t^m} \int_0^t \frac{(t-\tau)^{m-\alpha-1}}{\Gamma(m-\alpha)} u(x,\tau) d\tau, \quad t > 0. \tag{2.1}$$

Definition 2.2. Let $\alpha, t \in \mathbb{R}$ and $m-1 < \alpha < m \in \mathbb{N}, t > 0$, then

$$\begin{cases} \mathcal{D}_t^{\alpha} u(x,t) = \displaystyle\int_0^t \frac{(t-\tau)^{m-\alpha-1}}{\Gamma(m-\alpha)} \frac{\partial^m u(x,\tau)}{\partial \tau^m} d\tau, \\ \mathcal{D}_t^{\alpha} u(x,t) = \dfrac{\partial^m u(x,t)}{\partial t^m}, \alpha = m \in \mathbb{N}, \end{cases} \tag{2.2}$$

is called Caputo time fractional partial derivative for the function $u(x, t)$.

Definition 2.3. For $t = a + mh, h \neq 0, x > a, t > 0$ and $m-1 < \alpha < m \in \mathbb{N}$. The Grunwald–Letnikov time fractional partial derivative for a function $u(x, t)$ is defined as:

$$^{GL}\mathcal{D}_t^{\alpha} u(x,t) = \lim_{h \to 0} h^{-\alpha} \sum_{j=0}^{\frac{t-a}{h}} (-1)^j \binom{\alpha}{j} u(x, t-jh). \tag{2.3}$$

Theorem 2.1. *Let $\alpha_1, \alpha_2 \in \mathbb{R}$, such that $n-1 < \alpha_1 < n$, $m-1 < \alpha_2 < m$, $n \neq m$ for $n, m \in \mathbb{N}$. Then, in general*

$$\begin{cases} \mathcal{D}_t^{\alpha_1} \mathcal{D}_t^{\alpha_2} u(x,t) = \mathcal{D}_t^{\alpha_2} \mathcal{D}_t^{\alpha_1} u(x,t) = \mathcal{D}_t^{\alpha_1+\alpha_2} u(x,t), \\ \mathcal{D}_t^{\alpha_1} \mathcal{D}_t^{m} u(x,t) \neq \mathcal{D}_t^{m} \mathcal{D}_t^{\alpha_1} u(x,t). \end{cases} \tag{2.4}$$

Theorem 2.2. *Assume that the function $u(x, t)$ is infinitely α-differentiable function, for some $0 < \alpha \le 1$ at a neighborhood of a point (x, t_0), $t_0 > 0$. Then $u(x, t)$ has the fractional power series expansion:*

$$u(x,t) = \sum_{k=0}^{\infty} \frac{1}{\alpha^k k!} \left[\mathcal{D}_t^{k\alpha} u(x,t) \right]_{(x,t_0)} (t-t_0)^{k\alpha}, \tag{2.5}$$

where $\mathcal{D}_t^{k\alpha}$ denotes the application of time fractional partial derivative for k-times.

Definition 2.4. Let the function $u(x, t)$ be infinitely α-differentiable function for some $\alpha \in (0, 1]$. Then, the time fractional partial differential transform (TFPDT) of $u(x, t)$ is defined as:

$$_t\bar{U}_{\alpha}(x,k) = \frac{1}{\alpha^k k!} \left[\mathcal{D}_t^{k\alpha} u(x,t) \right]_{(x,t_0)}, t_0 > 0. \tag{2.6}$$

Definition 2.5. Let the function $u(x, t)$ be infinitely β-differentiable function for some $\beta \in (0, 1]$. Then, the space fractional partial differential transform (SFPDT) of $u(x, t)$ is defined as:

$$_x\bar{U}_{\alpha}(x,k) = \frac{1}{\beta^h h!} \left[\mathcal{D}_x^{h\beta} u(x,t) \right]_{x_0,t}, x_0 > 0. \tag{2.7}$$

where $\mathcal{D}_x^{h\beta}$ denotes the application of space-fractional partial derivative for h-times.

2.2. Conformable fractional partial derivatives

Definition 2.6. Given a function $u(x,t) : \mathbb{R} \times (0,\infty) \to \mathbb{R}$. Then, the conformable time fractional partial derivative of order α for a function $u(x, t)$ is defined as:

$$_t\mathcal{T}_{\alpha} u(x,t) = \lim_{\epsilon \to 0} \frac{u(x, t + \epsilon t^{1-\alpha}) - u(x,t)}{\epsilon}, \tag{2.8}$$

for all $t > 0$, $\alpha \in (0, 1]$.

Definition 2.7. Given a function $u(x,t) : (0,\infty) \times \mathbb{R} \to \mathbb{R}$. Then, the conformable space fractional partial derivative of order β for a function $u(x, t)$ is defined as:

$$_x\mathcal{T}_{\beta} u(x,t) = \lim_{\sigma \to 0} \frac{u(x + \sigma x^{1-\beta}, t) - u(x,t)}{\sigma}, \tag{2.9}$$

for all $x > 0$, $\beta \in (0, 1]$.

Theorem 2.3. *Let $\alpha \in (0, 1]$ and $u(x, t)$, $v(x, t)$ be α-differentiable functions at $(x,t) \in \mathbb{R} \times (0,\infty)$. Then*

(1) $_t\mathcal{T}_{\alpha}(au + bv) = a_t\mathcal{T}_{\alpha}u + b_t\mathcal{T}_{\alpha}v$,
(2) $_t\mathcal{T}_{\alpha}(t^p) = pt^{p-\alpha}, \forall p \in \mathbb{R}$,
(3) $_t\mathcal{T}_{\alpha}(\lambda) = 0$, for all constant functions $u(x,t) = \lambda$,
(4) $_t\mathcal{T}_{\alpha}(uv) = u_t\mathcal{T}_{\alpha}v + v_t\mathcal{T}_{\alpha}u$,
(5) $_t\mathcal{T}_{\alpha}(u/v) = \dfrac{v(_t\mathcal{T}_{\alpha}u) - u(_t\mathcal{T}_{\alpha}v)}{v^2}$,
(6) *If* u is differentiable with respect to t, then

$$_t\mathcal{T}_{\alpha} u(x,t) = t^{1-\alpha} \frac{\partial u(x,t)}{\partial t}.$$

Definition 2.8. Given a function $u(x,t) : \mathbb{R} \times [a,\infty) \to \mathbb{R}$ for $a \in \mathbb{R}, a \ge 0$. Then the conformable (left) time fractional partial derivative of order α for a function $u(x, t)$ is defined by

$$_t\mathcal{T}_{\alpha}^{a} u(x,t) = \lim_{\epsilon \to 0} \frac{u(x, t + \epsilon(t-a)^{1-\alpha}) - u(x,t)}{\epsilon}, \tag{2.10}$$

for all $t > a$, $\alpha \in (0, 1]$.

Definition 2.9. Given a function $u(x,t) : [b,\infty) \times \mathbb{R} \to \mathbb{R}$, for $b \in \mathbb{R}, b \ge 0$. Then the conformable (left) space fractional partial derivative of order β for a function $u(x, t)$ is defined by

$$_x\mathcal{T}_{\beta}^{b} u(x,t) = \lim_{\sigma \to 0} \frac{u(x + \sigma(x-b)^{1-\beta}, t) - u(x,t)}{\sigma}, \tag{2.11}$$

for all $x > b$, $\beta \in (0, 1]$.

Theorem 2.4. *Let $\alpha \in (0, 1]$ and u, v be α-differentiable at $(x,t) \in \mathbb{R} \times [a,\infty)$. Then*

(1) $_t\mathcal{T}_{\alpha}^{a}((t-a)^p) = p(t-a)^{p-\alpha}$ for all $p \in \mathbb{R}$,
(2) $_t\mathcal{T}_{\alpha}^{a}\left(e^{\lambda(\frac{(t-a)^{\alpha}}{\alpha}+x)}\right) = \lambda e^{\lambda(\frac{(t-a)^{\alpha}}{\alpha}+x)}$,
(3) $_t\mathcal{T}_{\alpha}^{a}\left(\dfrac{(t-a)^{\alpha}}{\alpha}\right) = 1$,
(4) *If* u is differentiable with respect to t, then

$$_t\mathcal{T}_{\alpha}^{a} u(x,t) = (t-a)^{1-\alpha} \frac{\partial u(x,t)}{\partial t}.$$

Theorem 2.5. *Let $\alpha \in (0, 1]$ and $u(x, t)$ to be $k-$times differentiable at $(x, t_0) \in \mathbb{R} \times (0,\infty)$. Then the $k-$times conformable time fractional partial derivative for a function $u(x, t)$ at the point (x, t_0) is given by*

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