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Mechanical analysis and bound of plasma chaotic system

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ABSTRACT

Plasma is normally investigated via fluid dynamics, and to investigate the force and energy underlying a plasma chaotic system, it is first transformed into a Kolmogorov-type system. This system describes a general form of fluid and forced-dissipative rigid body system. The vector field of the plasma chaotic system is decomposed into four types of torque: inertial torque, internal torque, dissipation, and external torque. The Hamiltonian energy transfer between kinetic energy and potential is discovered. The various combinations of these four types of torque are constructed to uncover the effect of each on the generation of the dynamic mode of the chaotic system. The physical functions of the whistler and dampening of the pump are identified in producing the different plasma dynamics. Aside from the torque effects, the rate of change of the Casimir function is also a key factor in characterizing the orbit behavior of the plasma system. Last, a supremum bound of the plasma chaotic attractor is proposed.

1. Introduction

Since the 50s, plasma physics has made much important progress and become a very active branch in physics. Plasma can be described by scattering wave and particle interaction theory from different aspects such as nonlinear oscillation, instability, turbulence, [1,2]. Furthermore, the waves in plasmas are an interconnected set of particles and fields that propagate in a periodically repeating fashion. Plasma as high-frequency damped waves has been studied in [3,4]; this system is nonlinear and dissipative. Plasma has been investigated via fluid dynamics, a case in point being the non-isentropic hydrodynamic models for two-carrier plasmas [5]. Applying hydrodynamics, analytic results for the efficiency of the transfer of latent heat to bulk motions of the plasma have been obtained [6].

From fluid dynamics, we find that the effect of force and energy transfers is important in dynamic systems. A whistler is assumed to propagate as energy along a magnetic field in the plasma, and it can excite (through force impacts) plasma waves and ion acoustic waves parametrically. Because energy is transferred to the waves that are not in resonance with the pump, the decay of these excitations needs to be taken into account. Because of interaction of the wave force and dampening and energy transfer of Plasma, the chaotic behavior is induced. In 1978, Pikovski, Rabinovich and Trakhtengerts introduced a plasma chaotic system [7]. When the pump amplitude is increased in the three-wave process, two of which are parametrically induced, a strange attractor appears.

Very recently, in regard to plasma chaos, there have been some research on the chaotic characteristics of nonlinear systems, such as control, synchronization, and its application [8–10]. However, these aspects cannot explain the mechanism or offer reasons for the generation of different dynamic modes. The force, interaction, and energy transfer have not been investigated for plasma. To explore the underlying dynamics, the mechanics of chaotic systems must be investigated. The impact of the damping force and whistler waves on plasma producing chaos can be uncovered by investigating the mechanics. The Kolmogorov system is a good starting point to investigate the mechanics of chaos because it has a general form that decomposes the force into inertial, internal, dissipative and external [11]. In 1991, a Kolmogorov system was presented describing the dissipative-forced dynamic system and hydro-dynamic instability beginning with the Hamiltonian function. Pasini and Pelino presented a unified treatment of the Kolmogorov and Lorenz systems thereby providing the forcing analysis of the Lorenz chaotic system [12]. Furthermore, Qi, et al., transformed the Qi four-wing chaotic system into a Kolmogorov-like system and employed this extended Kolmogorov system to perform a forcing analysis and energy cycling [13–15]. Liang and Qi transformed the Chen chaotic system into a Kolmogorov system to perform a forcing analysis and then interpreted the state of chaos as angular momentum [16,17]. In this regard, the Hamilto-

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nian function, Casimir function, and the Kolmogorov system provide a starting point to study the mechanisms underlying these chaotic systems.

The problem is that both the Qi four-wing chaotic system and the Qi chaotic system were built from numerical simulations instead of being derived from a physical model [18,19]. However, the plasma chaotic system is a real physical process in a magnetoactive nonisothermal plasma arising from the stochastic self-oscillations of waves amplitudes [20]. Therefore, a mechanical analysis of the plasma chaotic system is necessary in applications using the Kolmogorov model, which may help in the design and control of the system. Alternatively, plasma is a quasineutral, electrically conductive fluid, and therefore we may study the plasma system by transforming it into a Kolmogorov-like system.

The solution boundary is an important topic in chaos. Nevertheless, it is often quite difficult to find the bound for a chaotic attractor. In this regard, the Casimir function, like enstrophy or the potential vorticity in the context of fluid dynamics, is very useful in analyzing the stability conditions and the global description of the dynamical system [21,22]. The energetics of the Lorenz system using the Casimir function has already been studied [23]. We find that the evolution of the Casimir function is closely related to the bound of the plasma chaotic attractor.

Using the Kolmogorov form, this paper describes the decomposition of the vector field of torques into inertial, internal, dissipative and external contributions. Different dynamical modes arise from the different combination of torques. The functions of damping and whistler modes in producing different dynamics are analyzed. The supremum bound is analytically found using the rate of change of the Casimir function.

The rest of the paper is organized as follows: In Section 2, the original plasma chaotic system is transformed into a Kolmogorov system, and then the mechanics of the plasma chaotic system is analyzed. Section 3 investigates the mechanism underlying the different dynamic modes. In Section 4, the boundary of the chaotic attractor is established. A conclusion is given in Section 5.

2. Kolmogorov transformation of plasma chaotic system

In plasma an ionized gaseous substance becomes highly electrically conductive, so the long-range electric and magnetic fields dominate the behavior of the matter. By modeling the plasma as a fluid, an ordinary differential equation (ODE) was obtained from a partial differential equation (PDE) describing the plasma. The equations for the amplitudes of the interacting waves in magnetoactive plasma were obtained in standard fashion from the fluid equations describing the radio-frequency oscillations of an electron gas and from the kinetic equations describing the excitation of the radiation oscillation of the ion-acoustic wave. With $n, \mathbf{v}, \varphi$ denoting the radio-frequency density, velocity, and potential, respectively, the PDEs for this plasma system are [24,25]

$$\begin{aligned} &\frac{\partial \mathbf{v}}{\partial t} + [\mathbf{v}\ \boldsymbol{\Omega}_\mathbf{e}] - \frac{e}{m}\nabla\varphi + \mathbf{S}_1 = 0, \\ &\frac{\partial n}{\partial t} + n_0 \mathrm{div}\mathbf{v} + S_2 = 0. \end{aligned} \tag{1}$$

Here $\boldsymbol{\Omega}_\mathbf{e}$ is the cyclotron frequency, $\mathbf{S}_1 = (\mathbf{v}\nabla)v^d + (v^d\nabla)\mathbf{v}$, $S_2 = \mathrm{div}(n^d\mathbf{v} + nv^d)$; n^d, v^d are the low-frequency variations associated with the density and velocity of the electrons; and n_0 is their equilibrium density. Based on a dimensional analysis of system (1), Sturman [26] and Pikovskii, Rabinovich, and Trakhtengerts [7] obtained the simplest resultant equations by changing from natural variables n_k and n_x to normal (dimensionless) amplitudes a_k and b_x with

$$\begin{aligned} a_k &= \frac{\omega_p(2mn_0)^{1/2}}{k\omega_k^{1/2}}\left|\frac{\omega_p^2+\omega_H^2-2\omega_k^2}{\omega_k^2-\omega_H^2}\right|\frac{n_k}{n_0}, \\ b_k &= \left(\frac{2n_0T_e}{\Omega_x}\right)^{1/2}\frac{n_x}{n_0}, \end{aligned} \tag{2}$$

where n_k and n_x are the Fourier components of the variations of the electron densities. $\omega_p = (4\pi e^2 n_0 m^{-1})^{1/2}$, $\omega_H = eH(mc)^{-1}$, $\omega_k = \omega_p\omega_H(\omega_p^2+\omega_H^2)^{-1/2}\cos(\widehat{\mathbf{k}\,\mathbf{z}})$, $\mathbf{k}$ is the plasma wave vector, $\mathbf{z}\|\mathbf{H}$, $\mathbf{H}$ is the magnetic field, T_e is electron temperature, Ω_x is ion sound frequency.

As the statements in [27], Pikovskii et al. [7] found that a whistler can destabilize magnetoactive plasma using exciting the lower hybrid wave together with the ion acoustic wave (the longitudinal compression wave in the ion density of a plasma). Specifically, the whistler at frequency ω_q excites a plasma wave at frequency ω_k and the ion acoustic wave at frequency $\Omega_x = \omega_q - \omega_k$. We call a_k the normal amplitude of the wave at frequency ω_k and b_x the normal amplitude of the ion acoustic wave. As a result of the decay of these excitations, at least a third synchronous wave is produced (of normal amplitude a_{k_1}). The dynamics of the interaction of wave in the plasma propagating parallel to the magnetic field with ion acoustic wave and plasma oscillation near lower hybrid resonance is described as follow [7,27]:

$$\begin{aligned} \dot{a}_k &= -b_x a_{k_1} - \nu_1 a_k + hb_x^*, \\ \dot{b}_x &= a_k a_{k_1}^* - \nu_2 b_x + ha_k^*, \\ \dot{a}_{k_1} &= a_k b_x^* - a_{k_1}, \end{aligned} \tag{3}$$

where $a_k^*, b_x^*, a_{k_1}^*$ are conjugate of a_k, b_x, a_{k_1}. h is proportional to the amplitude of pump, which is the electric field of the whistler; ν_1 and ν_2 are the damping decrements of the excited hybrid and acoustic waves normalized to the damping of the synchronous wave. We study the dynamics of real wave amplitudes a_k, b_x, a_{k_1} which can be shown that they correlate as $t \to \infty$, i.e.,

$$\begin{aligned} \dot{a}_k &= -b_x a_{k_1} - \nu_1 a_k + hb_x, \\ \dot{b}_x &= a_k a_{k_1} - \nu_2 b_x + ha_k, \\ \dot{a}_{k_1} &= a_k b_x - a_{k_1}. \end{aligned} \tag{4}$$

To discover the physical analogue of the state variables and the mechanics of the above system, we introduce the Kolmogorov system, which is a generalized Euler equation with dissipation and generalized external torque. The Kolmogorov system is a general model for a class of chaotic system that is useful in analyzing force and energy transfer [13]. The Kolmogorov system is written [11]

$$\dot{\mathbf{x}} = \{\mathbf{x}, H\} - \Lambda\mathbf{x} + \mathbf{f}, \tag{5}$$

where $\mathbf{x} = [x_1, x_2, x_3]^\mathrm{T}$, $\{\mathbf{x}, H\}$ representing the Lie–Poisson bracket of the Hamiltonian function of the system, denoted H, Λ is a positive definite diagonal matrix, $-\Lambda\mathbf{x}$ represents the dissipation, and the last term $\mathbf{f}$ represents generalized external torque (or force).

Next, we establish an analogy between the plasma chaotic system and the Kolmogorov system. We have to identify the Lie–Poisson bracket, the dissipative terms, and external torque parts, respectively. The Euler equation for an incompressible fluid or a free rigid body is written [28]

$$\dot{\mathbf{x}} = \{\mathbf{x}, K\} = \mathbf{x} \times \Pi\mathbf{x}, \tag{6}$$

where

$$K = \frac{1}{2}\left(\Pi_1 x_1^2 + \Pi_2 x_2^2 + \Pi_3 x_3^2\right) \tag{7}$$

is the kinetic energy, $\Pi = diag(\Pi_1 \quad \Pi_2 \quad \Pi_3)$, $\Pi_i = I_i^{-1}$, I_i is the principle moment of inertia for the group SO(3), and x_i the angular momentum satisfying $x_i = I_i\omega_i$, and ω_i the angular velocity. In