



Frontiers

A mathematical analysis of Pine Wilt disease with variable population size and optimal control strategies



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ABSTRACT

The present paper describes the dynamics of the Pine Wilt disease with variable population size. The basic mathematical results for the model are presented. The stability analysis of both the disease-free and endemic cases are presented whenever \mathcal{R}_0 lesser or greater than one, respectively. Further, an optimal control problem is formulated and the necessarily involved results are presented. Moreover, the numerical simulation of the optimal control problem with suggested control strategies for the possible eliminations of the infection in the pine trees population is presented.

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1. Introduction

Mathematical models have played important role in the dynamics of infectious diseases. The main causes of infectious diseases are the pathogens that are bacteria, viruses, and protozoa. These pathogens can reach to the living healthy organism by the biological agents, a vector that become the cause of the disease. The vector class includes insects, ticks, etc., that carry the pathogen from infective one to a healthy individual. The infectious diseases that caused by the vector is known as vector-borne disease. The infectious diseases do not only occur in the human population, but it can also affect the population of trees. Pine Wilt disease (PWD) is a pine tree disease that caused by the pine-wood nematode *Bursaphelenchus xylophilus* [1,2]. It is considered a dramatic disease because it kills affected trees within a few weeks to a few months. The pinewood nematode (*Bursaphelenchus xylophilus*) that causes the death of the trees is microscopic. Mostly the plant parasites are in connection with plant roots, but in the case of pinewood nematode, it is found in the aboveground parts of the tree. The vector for the pine wilt trees is pine sawyer beetle (or *Monochamus alternatus*). On healthy pine trees, the beetle spread their nematode

from space. This pinewood disease caused by the pinewood nematode was firstly investigated in 1905 in Japan [3]. In the 1980s, the Pine Wilt disease epidemic had spread to other Asian countries, such as Hong Kong, Korea, China and Taiwan. In 1999, PWD reached to Portugal [4–6]. The Pine Wilt disease regarded a serious threat to pine forest worldwide. Nematodes kill the tree by feeding on the cells surrounding the resin ducts. This causes the resin to leak into the tracheid's, resulting in "tracheid cavitation" or air pockets in the water transport system. Just as a person cannot drink through a straw with holes in it, the tree cannot move water upward and consequently wilts and dies. Pine sawyer beetles (*Monochamus* spp.) vector the nematodes. The nematodes hitch a ride with the beetles and thus move from tree to tree as the beetles feed on the young shoots of pine trees [7].

The first observable symptom is the lack of resin exudation of bark wounds. The foliage then becomes pale green, then yellow, and finally reddish brown as the tree succumbs to the disease. It is well known that there are three transmission pathways of PWD. One occurs during maturation feeding when the nematode is transferred from insect vectors to healthy pine trees via the insect feeding on wounds [8]. The second occurs during oviposition of the mature female on dead, dying, or recently cut pine trees via the oviposition wounds [9]. The third occurs during mating, that is, as mature males search for females in bark wounds, such as the

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oviposition wounds. This is referred to as horizontal transmission [10].

In literature, many articles have been presented on the dynamics of PWD. In [11] presented a mathematical model on PWD and discussed the stability of Pine Wilt disease with optimal control. He divided the population of pine trees into two categories that are susceptible pine trees and infected pine trees, whereas the population of the vector (beetles) is divided into two classes; susceptible vector and infected vector. K. S. Lee and A. A. Lashari [12] introduced a mathematical model that incorporated the exposed class in the pine tree population. A detail discussion has been made on the stability and optimal control of PWD. M. Ozair presented a mathematical model of the dynamics of PWD by dividing the host pine trees and vector beetles into susceptible and infected classes with nonlinear incidence and horizontal transmission [13]. K. S. Lee and D. Kim introduced a mathematical model that describes the dynamics of PWD by presenting its global stability with nonlinear incidence rates [14].

In this paper, we introduced a more advanced model that divides the total population of host pine trees into three classes, namely, $S_H(t)$ -susceptible pine trees, $E_H(t)$ -exposed pine trees and $I_H(t)$ -infected pine trees and the population of vector beetles is divided into three categories, namely, $S_V(t)$ -susceptible beetle, $E_V(t)$ -exposed beetle and $I_V(t)$ -infected vector beetle. We first present the detail mathematical study of the model that is the local and global asymptotical stability. Then, we applying the optimal control technique to minimize the population of infected pine trees, susceptible vector and infected vector (beetles). Different control strategies will be used for the elimination of infection in the population of pine trees. In the next, section “Problem Formulation” we give a detailed analysis of the mathematical formulation of PWD. In the section “Basic results” we reduce the model. The stability and equilibria are described in the section “Model Stability and Equilibria”. The global stability of both the cases, disease-free state and endemic state is presented in section “Global stability of the model”. Further, we apply the optimal control technique in section “Optimal control problem”. The optimal control problem solution is presented in section “Optimal control solution” while the numerical results and conclusion are presented in sections “Numerical results” and “Conclusion” respectively.

2. Problem formulation

To model the Pine Wilt disease PWD (pine trees and vector beetles), we consider the total size of the Pine Wilt disease (PWD) by $N_H(t)$, which further subdividing into three categories: $S_H(t)$ -susceptible pine trees, $E_H(t)$ -exposed pine trees (which are not yet infectious) and $I_H(t)$ -infected pine trees at any time t . Here, in the population of PWD, we don't consider the class $R_H(t)$, because, the infected trees die due to the disease within a year. The population size of PWD can be written as $N_H(t) = S_H(t) + E_H(t) + I_H(t)$.

The population of susceptible pine trees S_H are generated by Π_H and decreased by the effective contact rate $\frac{\beta_1 S_H I_V}{N_V}$, where β_1 is the probability at which the infected beetles transmit nematodes by means of contact, further the population of susceptible pine trees are decreased by the contact rate $\frac{\beta_2 \psi S_H I_V}{N_V}$, where β_2 the rate at which the infected beetles transmit nematodes by oviposition and the parameter ψ measure the probability of natural mortality rate of susceptible pine trees instead of nematode. Moreover, the natural death rate of pine trees decrees further the population of pine trees. The above discussion leads to the following differential equation,

$$S'_H(t) = \Pi_H N_H - \frac{\beta_1 S_H I_V}{N_V} - \frac{\beta_2 \psi S_H I_V}{N_V} - \gamma_H S_H. \tag{1}$$

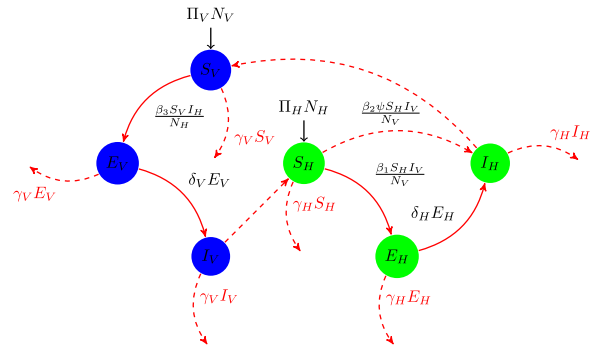


Fig. 1. Flow diagram for the Pine Wilt disease PWD.

The population of exposed pine trees, E_H , are increased by the contact rate $\frac{\beta_1 S_H I_V}{N_V}$, and decreased by the natural death rate γ_H and the transfer rate of exposed pine trees getting infection by the pine trees at a rate δ_H . Thus, its differential equation can be written as

$$E'_H(t) = \frac{\beta_1 S_H I_V}{N_V} - (\gamma_H + \delta_H) E_H. \tag{2}$$

The population of infected pine trees, I_H , are increased by the transfer rate δ_H and the contact rate $\frac{\beta_2 \psi S_H I_V}{N_V}$, while decreased by the natural death rate γ_H . Thus, its differential equation can be written as

$$I'_H(t) = \frac{\beta_2 \psi S_H I_V}{N_V} + \delta_H E_H - \gamma_H I_H. \tag{3}$$

We denote the population of vector (beetles) by $N_V(t)$, subdividing into three classes, namely, $S_V(t)$ -the susceptible adult beetles (not carrying the pinewood nematode), $E_V(t)$ -the exposed adult beetles (which are not yet infected) and $I_V(t)$ -infected beetles (have the ability to carry the pinewood nematode), that is, adult beetles from dead pine trees at any time t . The total population of vector (beetles) is $N_V(t) = S_V(t) + E_V(t) + I_V(t)$.

The population of susceptible beetles is increased by the recruitment rate Π_V , and by the effective contact rate $\frac{\beta_3 S_V I_H}{N_H}$, where β_3 is the probability of getting infected with the pinewood nematode when the beetle emerge out from the infected class of pine trees. Further, the population of susceptible beetles is decreased by the natural death rate γ_V . Thus, its differential equation can be written as

$$S'_V(t) = \Pi_V N_V - \frac{\beta_3 S_V I_H}{N_H} - \gamma_V S_V. \tag{4}$$

The population of exposed beetles is increased by the effective contact rate $\frac{\beta_3 S_V I_H}{N_H}$ and decreased by the transfer rate δ_V (from E_V to I_V) and the natural death rate γ_V . Thus, its differential equation can be written as

$$E'_V(t) = \frac{\beta_3 S_V I_H}{N_H} - (\gamma_V + \delta_V) E_V. \tag{5}$$

The population of infected beetles is increased by the transfer rate of exposed individuals shown by parameter δ_V and decreases by the natural death rate γ_V . Thus, its differential equation can be written as

$$I'_V(t) = \delta_V E_V - \gamma_V I_V. \tag{6}$$

The complete transfer of each flow of parameter and variable are presented in Fig. 1. The total dynamics of pine trees is given by

$$\frac{dN_H}{dt} = \Pi_H N_H - \gamma_H N_H = (\Pi_H - \gamma_H) N_H. \tag{7}$$

The exact solution of Eq. (7) is given by

$$N_H(t) = N_H(0) e^{(\Pi_H - \gamma_H)t}.$$

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