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Robust adaptive beam-forming optimization method based on diagonal-loading and MSE criterion

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ABSTRACT

Smart antenna can effectively suppress multipath interference, co-channel interference, and improves the transmission quality of signal and the utilization of spectrum, so it is widely applied in wireless communication network. To explore the optimization problem about smart antenna receiving array and sensor network which exists in radar, sonar and other systems, here we presents a kind of adaptive beam-forming algorithm based on diagonal-loading and mean square error (MSE) criterion. Such a novel algorithm could give the optimal solution of weight direction vector, and at the same guarantees its own robustness. Furthermore, it also possesses the advantage of shortening the convergence time of weight direction vector, and decreasing the sensitive issue of model error in high SNR environment. In our simulation experiments, it is shown that the proposed algorithm improves the performance of receiver network in sonar system, and to a certain extent achieves the signal optimization.

1. Introduction

Underwater Acoustic Imaging technology has been frequently used in many military and civilian fields, including underwater target detection, sunken ships searching, underwater landscape drawing, the black box salvaging, etc. In order to achieve a certain distance high-resolution acoustic imaging of target object under the water, it seems very constructive and worth great attention to study stable imaging technology for complex sounds of the underwater environment, with certain imaging frame rate and action distance [1]. Hence, underwater acoustic imaging adaptive beam-forming algorithm plays a significant role, which not only involves the robustness of algorithm, but also improves the output SNR of system at the cost of low calculation.

Adaptive beam-forming technology changes the pattern of array by adjusting the weight vector, to make main lobe aim at desired signal, and side lobes aim at interfering signal, so as to increase the output SNR, and achieve best reception under certain criteria.

LMS adaptive beam-forming algorithm is a beam forming method, which has simple structure, low complexity, easy for implementation and high stability. Because of its slow convergence, it suffers from some certain restriction when applied in engineering applications. Addressed on this problem, several adjustment strategies were proposed by the research community, such as instantaneous error, right forward prediction vector, and smoothing gradient vector, and propose variable step size LMS algorithm, so as to balance convergence speed and algorithm disorders [2]. Although these algorithms show better performance on balancing convergence speed and disorders than the classical LMS algorithm, their ability to cope with the mutation problem were decreased.

As one of the criteria to determine the best reception, Widrow et al. proposed with the mean square error (MSE) performance measure [3]. Wiener and Hopf deduced the optimal Wiener solution [4]. Based on MSE criterion, classic LMS algorithm get optimal weight vector, by utilizing optimization methods such as the steepest descent method, and accelerating gradient method.

Based on the above discussion, we propose a robust adaptive beam-forming algorithm based on MSE criterion and diagonal-loading technique (Some scholars have studied the adaptive beam-forming algorithm based on the coupling coefficient [5–8]). In this algorithm, it reconstructs sampling covariance matrix by artificially injecting white noise into the sampling covariance matrix diagonal line, which is named as diagonal-loading. Then it uses the formula of matrix inversion to avoid the calculation of matrix inversion and iteration, and converts the diagonal-loading coefficient into LMS algorithm step factor function. Simulation results show that this algorithm can effectively reduce convergence time, and have better robustness at high and low SNR environment.

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2. The model of signal

The proposed algorithm is based on the linear array of flat space. The following conditions should be satisfied:

(1) Array elements are equally spaced
(2) Narrowband assurance: This ensures that all array elements almost received the signal simultaneously, and the signal envelope that is received between the array elements does not change.
(3) Ignore the mutual coupling between array elements.
(4) The number of signals to be smaller than the number of array elements, and the array of the received signal DOA all different from each other.
(5) Plane wave assumptions: The distance from the sources to array is much larger than the distance between array elements, so that all incoming signals to the array can be approximated as plane waves.

2.1. Parameters declaration

M	The number of array elements;
d	The distance of the array, $d = \lambda/2$;
λ	The received signal wavelength of array elements;
L	The number of the echo sources;
θ_i	The DOA of the echo sources, $\theta_i = \{\theta_1, \theta_2, ..., \theta_L\}$;
$d(t)$	The reference signal.

2.2. Signal receiving model

In the first array element of the array as a reference, the value of the k-th sample snapshot at a sampling point $m(1 \le m \le M)$ can be expressed as

$$x_m(k) = \sum_{i=1}^{L} s_i(k) \exp\left[j\frac{2\pi}{\lambda}(m-1)d\sin\theta_i\right] + n_m(k) \tag{1}$$

Here, $n_m(k)$ represents the noise on the m-th array element and $s_i(k)$ means each echo source's base-band signal at the reference point. Then we can get the model of array output in the time domain

$$y(t) = \boldsymbol{\omega}^T \mathbf{x}(t) \tag{2}$$

The expression after sampling is

$$y(k) = \boldsymbol{\omega}^T \mathbf{x}(k) \tag{3}$$

Here, $\mathbf{x}(k)$ represents the input of array ($\mathbf{x}(k) = [\mathbf{x}_1(k), \mathbf{x}_2(k), ...\mathbf{x}_M(k)]^T$), and $\boldsymbol{\omega}$ represents the weight vector on the array elements ($\boldsymbol{\omega} = [\omega_1, \omega_2, ..., \omega_M]^T$).

2.3. MSE performance metric

The error between reference signal and actual output signal can be represented by the equation that followed:

$$\varepsilon(t) = d(t) - y(t) = d(t) - \boldsymbol{\omega}^T \mathbf{x}(t) \tag{4}$$

Then, square this equation, we get:

$$\varepsilon^2(t) = d^2(t) - 2d(t)\boldsymbol{\omega}^T \mathbf{x}(t) + \boldsymbol{\omega}^T \mathbf{x}(t)\mathbf{x}^T(t)\boldsymbol{\omega} \tag{5}$$

On both sides of the equation, we seek mathematical expectation:

$$E\{\varepsilon^2(t)\} = \overline{d^2(t)} - 2\boldsymbol{\omega}^T \mathbf{R}_{xd} + \boldsymbol{\omega}^T \mathbf{R}_{xx}\boldsymbol{\omega} \tag{6}$$

In the above formula: $\overline{d^2(t)}$ represents the mathematical expectation of $d(t)$; $\mathbf{R}_{xd}$ represents the cross-correlation matrix of reference signal and actual output signal, that is $\mathbf{R}_{xd} = E\{d(k)\mathbf{x}^T(k)\}$. The self-correlation matrix is expressed by $\mathbf{R}_{xx} = E\{\mathbf{x}_s \mathbf{x}_s{}^H\}$.

Let $\overline{d^2(t)}$=S, thus we get the formula base on the MSE performance metric:

$$E\{\varepsilon^2(t)\} = S - 2\boldsymbol{\omega}^T \mathbf{R}_{xd} + \boldsymbol{\omega}^T \mathbf{R}_{xx}\boldsymbol{\omega} \tag{7}$$

Select an appropriate weight vector $\boldsymbol{\omega}$, we can minimize $E\{\varepsilon^2(t)\}$. Eq. (7) is a quadratic function of the weight vector $\boldsymbol{\omega}$, with its extreme value being a minimum. Then, using gradient algorithm, we could get the optimal solution $\boldsymbol{\omega}_{opt}$ of above formula, which satisfies:

$$\boldsymbol{\omega}_{opt} = \mathbf{R}_{xx}^{-1}\mathbf{R}_{xd} \tag{8}$$

3. The derivation of algorithm

In terms of beam forming algorithm, LMS algorithm as a fixed step size LMS algorithm, its iterative formula of the weight vector can be expressed by the following equation

$$\boldsymbol{\omega}(k+1) = \boldsymbol{\omega}(k) - \mu\nabla(k) \tag{9}$$

In order to overcome the covariance matrix inversion operation, LMS algorithm uses the steepest descent method to solve formula (9), which can get the iterative formula of LMS algorithm as following:

$$\boldsymbol{\omega}(k+1) = \boldsymbol{\omega}(k) + \mu\mathbf{x}(k)e^*(k) \tag{10}$$

Here, μ represents the step factor which controlling adaptive rate. The range of step factor μ is discussed and analyzed in literature [9,10]. Let's assume that it satisfies: $0 < \mu < \frac{1}{\gamma_{\max}}$. When the number of iterations increases infinitely, we can prove that the weight vector expectations converge to the Wiener solution. This algorithm is improved by several different methods [11–13], in which the calculations of the eigenvalues decomposition and inversion of covariance matrix could be avoided. However, the optimal weight vector values still need to be calculated by iteration. The adaptive beam-forming method proposed in this paper is based on the LMS algorithm, and by avoiding the calculation of iteration, the proposed algorithm makes it faster to convergence to the optimal value.

To enhance the robustness of adaptive beam-forming, the diagonal-loading technique has been used to suppress pattern distortion. According to the signal model mentioned in the second part, the actual sampling covariance matrix $\mathbf{R}_{xx}$ is replaced by an estimate of the k-th signal sampled:

$$\hat{\mathbf{R}}_{xx} = \frac{1}{K}\sum_{k=1}^{K}\mathbf{x}(k)\mathbf{x}^H(k) \tag{11}$$

Thus, formula (8) can be expressed as:

$$\boldsymbol{\omega}_{\mathrm{opt}} = \hat{\mathbf{R}}_{xx}^{-1}\mathbf{R}_{xd} \tag{12}$$

Here, the diagonal loading is applied to the weight vector calculation of the algorithm. We can get

$$\tilde{\mathbf{R}}_{xx} = (\xi\mathbf{I} + \hat{\mathbf{R}}_{xx}) \tag{13}$$

Lemma. Assume matrix $\mathbf{A} \in \mathbf{C}^{n \times n}$, and its inverse matrix exists, $\mathbf{x}$ and $\mathbf{y}$ are $n \times 1$ vectors, which makes $(\mathbf{A} + \mathbf{x}\mathbf{y}^H)$ reversible, that is

$$(\mathbf{A} + \mathbf{x}\mathbf{y}^H)^{-1} = \mathbf{A}^{-1} - \frac{\mathbf{A}^{-1}\mathbf{x}\mathbf{y}^H\mathbf{A}^{-1}}{1 + \mathbf{y}^H\mathbf{A}^{-1}\mathbf{x}} \tag{14}$$

And it can be extended to the inverse formula of matrix sum, that is

$$\begin{aligned}(\mathbf{A} + \mathbf{UBV})^{-1} &= \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{UB}(\mathbf{B} + \mathbf{BVA}^{-1}\mathbf{UB})^{-1}\mathbf{BVA}^{-1} \\ &= \mathbf{A}^{-1} - \mathbf{A}^{-1}\mathbf{U}(\mathbf{I} + \mathbf{BVA}^{-1}\mathbf{U})^{-1}\mathbf{BVA}^{-1}\end{aligned} \tag{15}$$