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Traveling wave fronts of a single species model with cannibalism and nonlocal effect

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ABSTRACT

In this paper, we present a single species model with cannibalism and nonlocal effect. The existence of traveling wave fronts connecting the equilibrium 0 to the equilibrium $\frac{Kr}{r+Kh}$ is proved when the wave speed $c \geq 2\sqrt{r}$.

1. Introduction

Mathematical model is an important and effective tools of solving practical problems. From the perspective of mathematical dynamics, it is enable to analyze some biological phenomena more precisely. For instance, single-species models can reveal intraspecific interactions and relationships between the species and the environment. In the long run of course, logistic growth is relatively reasonable [1]. And intraspecific interactions are competition, cannibalism and altruism. In reality, the time delay and spatial diffusion is universal [2–8]. Specifically, time delay is common and inevitable in nature which denotes resource regeneration time, individual mature period, lactation time, feedback time and so on. Murray proposed the following differential delay equation [1]:

$$\frac{dN}{dt} = rN(t)\left[1 - \frac{N(t-T)}{K}\right]$$

where r, K and T are positive constants. It means that the regulatory effect depends on the population at an earlier time $t-T$, rather than that at t. In the model, time delay T can be interpreted as maturity period.

Moreover, explanation of the diffusion is that every individual walks in a random way instead of standing still [9–12]. And under the assumption of the unbiased motion, we have derived the mathematical formula with regarding to diffusion by the method of probability analysis [1,13–15]. In general, Laplace term can represent species diffusion, whereas, which only describes small scale diffusion, that is local diffusion. As is well known, the individual is moving all the time. So, for the case of the individual being at x at time t, we must take account of the fact that it may not have been at any given previous time $t-T$. Britton firstly proposed the idea of combining time delay with the weighted average of space, that is adding the convolution of time and space to the reaction-diffusion equation with delay [16–18], and the spatio-temporal delay is put in the place of nonlinear term. Britton proposed the reaction-diffusion model with nonlocal delay:

$$\frac{\partial}{\partial t}u(x,t) = \triangle u(x,t) + u(x,t)[1 + au(x,t) - (1+a)(h*u)(x,t)],$$
$$x \in \Omega,\ t > 0,$$

where

$$(h*u)(x,t) = \int_{-\infty}^{t}\int_{\Omega} h(x-y,t-s)u(y,s)dyds.$$

Traveling wave solutions of reaction-diffusion equation can depict the long term behaviors of species. And it's known that traveling wave front is one of the traveling wave solutions. There have been many scholars studying the traveling wave fronts [19–21].

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In this paper, we are concerned with the following reaction diffusion equation with nonlocal effect and logistic growth and cannibalism:

$$\frac{\partial u}{\partial t} = u_{xx} + ru\left[1 - \frac{(f*u)(x,t)}{K}\right] - hu^2 \quad in\ \mathbb{R}\times(0,\infty), \tag{1}$$

where r represents the intrinsic growth rate of the species, K represents the capacity of the environment. As for the term hu^2 signifying intraspecific cannibalism, speaking specifically, intraspecific cannibalism defined as intraspecific predation is a widespread phenomenon in a variety of animals. From the view of the individual level, it results in an increase in death rate. At the population level, cannibalism has the potential of regulating population size. And the cannibalism term can interpret as the situation that an individual encounters with another by interaction with constant cannibalism rate h. More details can be found in Refs. [22–25]. The kernel $f(x,t) \in L^1(\mathbb{R}\times(0,\infty))$ satisfies $f(x,t) \geq 0$, $\int_{-\infty}^{t}\int_{-\infty}^{+\infty} f(y,s)dyds = 1$ and

$$(f*u)(x,t) = \int_{-\infty}^{t}\int_{-\infty}^{+\infty} f(x-y,t-s)u(y,s)dyds.$$

System (1) has two constant equilibria

$$u_1 := 0, \qquad u_2 := \frac{Kr}{r+Kh}.$$

Without loss of generality, we set the kernel function as the following form: $f(x,t) = \frac{1}{\sqrt{4\pi\rho}}e^{-\frac{x^2}{4\rho}}\delta(t)$, where ρ signifies nonlocal effect [26].

In this paper, we show traveling wave fronts of system (1). More specifically, the existence of traveling wave fronts with the wave speed $c \geq 2\sqrt{r}$ is proved from the equilibrium 0 to the equilibrium $\frac{Kr}{r+Kh}$.

2. Existence of traveling wave fronts

In this section, we firstly prove the existence of traveling wave fronts on $\mathbb{R}$ with $c > 2\sqrt{r}$ by using the method of sub- and supersolutions [26–28]. Next, for the case of $c = 2\sqrt{r}$, we use some priori estimates [19–21] and the Leray–Schauder degree theory [29] to find a solution on a finite interval and then we set $a \to +\infty$ to obtain a solution of system (3) on $\mathbb{R}$.

Considering the traveling wave fronts of system (1), we let $u(x,t) = \varphi(x+ct) \triangleq \varphi(t)$, then put it into system (1)

$$c\varphi'(t) = \varphi''(t) + r\varphi\left[1 - \frac{(f*\varphi)(t)}{K}\right] - h\varphi^2, \quad t\in\mathbb{R}. \tag{2}$$

where $(f*\varphi)(t) = \int_0^{+\infty}\int_{-\infty}^{+\infty} f(y,s)\varphi(t-y-cs)dyds = \int_{-\infty}^{+\infty}\frac{1}{\sqrt{4\pi\rho}}e^{-\frac{y^2}{4\rho}}\varphi(t-y)dy$.

Making variable substitution $\psi(t) = \frac{\varphi(t)}{\frac{Kr}{r+Kh}}$, (2) is changed into the following equation:

$$c\psi'(t) = \psi''(t) + r\psi\left[1 - \frac{r}{r+Kh}(f*\psi)(t) - \frac{Kh}{r+Kh}\psi\right], \quad t\in\mathbb{R}. \tag{3}$$

where $(f*\psi)(t) = \int_{-\infty}^{+\infty}\frac{1}{\sqrt{4\pi\rho}}e^{-\frac{y^2}{4\rho}}\psi(t-y)dy$.

Theorem 1. *For any $c \geq 2\sqrt{r}$, there exists a traveling wave fronts $u(x,t) = \varphi(x+ct) = \varphi(t)$ satisfying system (2) which means that $\psi(t)$ satisfies system (3) under the asymptotic boundary conditions*

$$\lim_{t\to-\infty}\psi(t) = 0, \qquad \lim_{t\to+\infty}\psi(t) = 1. \tag{4}$$

2.1. Existence of traveling wave fronts on $\mathbb{R}$ with $c > 2\sqrt{r}$

In this part, we prove the first part of Theorem 2.

Denote $(\mathcal{F}\psi)(t) := r\psi[1 - \frac{r}{r+Kh}(f*\psi)(t) - \frac{Kh}{r+Kh}\psi]$.

Lemma 1. *There exists a γ_1 such that $(\mathcal{F}\psi_2)(t) + \gamma_1\psi_2(t) \geq (\mathcal{F}\psi_1)(t) + \gamma_1\psi_1(t)$ on $t\in\mathbb{R}$, for any $\psi_1(t), \psi_2(t)$ which satisfies*

(i) *$0 \leq \psi_1(t) \leq \psi_2(t) \leq 1$;*
(ii) *$e^{\gamma_1 t}[\psi_2(t) - \psi_1(t)]$ and $e^{-\gamma_1 t}[\psi_2(t) - \psi_1(t)]$ is respectively increasing and decreasing.*

Proof. Since $\psi_1(t)$, $\psi_2(t)$ satisfies (i),(ii), we have $(\mathcal{F}\psi_2)(t) - (\mathcal{F}\psi_1)(t) = r\psi_2(t)[1 - \frac{r}{r+Kh}(f*\psi_2)(t) - \frac{Kh}{r+Kh}\psi_2(t)] - r\psi_1(t)[1 - \frac{r}{r+Kh}(f*\psi_1)(t) - \frac{Kh}{r+Kh}\psi_1(t)] \geq -\frac{r}{r+Kh}[\psi_2(t) - \psi_1(t)] - \frac{r^2}{r+Kh}[(f*\psi_2)(t) - (f*\psi_1)(t)] - \frac{r}{r+Kh}[\psi_2(t) - \psi_1(t)] - \frac{r^2}{r+Kh}\{\int_{-\infty}^{+\infty}\frac{1}{\sqrt{4\pi\rho}}e^{-\frac{y^2}{4\rho}}[\psi_2(t-y) - \psi_1(t-y)]dy\} \geq -\frac{r+2r^2e^{\rho\gamma^2}}{r+Kh}[\psi_2(t) - \psi_1(t)] \geq -\gamma_1[\psi_2(t) - \psi_1(t)]$.

We take $\gamma_1 > \frac{r+2r^2e^{\rho\gamma^2}}{r+Kh}$.

The following equation $\triangle_c(\lambda) = \lambda^2 - c\lambda + r = 0$, yields two roots

$$0 < \lambda_1 = \frac{c - \sqrt{c^2-4r}}{2} < \lambda_2 = \frac{c - \sqrt{c^2+4r}}{2}.$$

Define

$$q(t) = \frac{1}{1+\alpha e^{-\lambda_1 t}}, \qquad p(t) = \max\{e^{\lambda_1 t}(1 - Me^{\varepsilon t}), 0\}$$

where $0 < \varepsilon < \lambda_1$, $\alpha < \frac{\lambda_1}{2(\gamma+\lambda_1)}$, and $M \geq \max\{\frac{1}{\alpha}, -\frac{r(1+e^{\frac{\lambda_1^2}{16\rho}})}{\triangle_c(\lambda_1+\varepsilon)}, \frac{1}{1-\alpha}\}$. □

Lemma 2. *There are a supersolution $q(t)$ and a subsolution $p(t)$ of system (3) such that $q(t)$ is increasing on $t\in\mathbb{R}$, and $q(t) \geq p(t)$, $q(t)$ satisfies (4) as well.*

Proof. We divide our discussions into two steps.

Step 1. $q(t)$ is a supersolution and $p(t)$ is a subsolution of system (3).

Suppose distribution function

$$F(y,\rho) = \int_{-\infty}^{y}\frac{1}{\sqrt{4\pi\rho}}e^{-\frac{\xi^2}{4\rho}}d\xi,$$

then $F(-\infty,\rho) = 0$, $F(0,\rho) = \frac{1}{2}$, $\frac{\partial}{\partial y}F(y,\rho) = \frac{1}{\sqrt{4\pi\rho}}e^{-\frac{y^2}{4\rho}}$.

Since $\lim_{y\to-\infty}e^{-\lambda_1 y}F(y,\rho) = 0$, we obtain by partial integration:

$$\int_{-\infty}^{0} e^{-\lambda_1 y}F(y,\rho)dy = -\frac{1}{\lambda_1}\int_{-\infty}^{0}F(y,\rho)d(e^{-\lambda_1 y})$$
$$= -\frac{1}{2\lambda_1} + \frac{1}{2\lambda_1}e^{\rho\lambda_1^2} + \frac{1}{\lambda_1\sqrt{\pi}}e^{\rho\lambda_1^2}\int_0^{\sqrt{\rho}\lambda_1}e^{-y^2}dy.$$

Define

$$G_-(y,\rho) = \int_{-\infty}^{y}e^{-\lambda_1\xi}F(\xi,\rho)d\xi,$$
$$G_+(y,\rho) = \int_{-\infty}^{y}e^{\lambda_1\xi}F(\xi,\rho)d\xi,$$

then,

$$G_-(-\infty,\rho) = 0,$$
$$G_-(0,\rho) = -\frac{1}{2\lambda_1} + \frac{1}{2\lambda_1}e^{\rho\lambda_1^2} + \frac{1}{\lambda_1\sqrt{\pi}}e^{\rho\lambda_1^2}\int_0^{\sqrt{\rho}\lambda_1}e^{-y^2}dy,$$
$$G_+(-\infty,\rho) = 0.$$