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Analysis of epidemic spreading process in multi-communities

Peican Zhu[a,b], Xing Wang[a], Qiang Zhi[a], Jiezhong Ma[a], Yangming Guo[a,*]

[a] *School of Computer Science, Northwestern Polytechnical University (NWPU), Xi'an, Shaanxi, 710072, China*
[b] *The Centre for Multidisciplinary Convergence Computing (CMCC), Northwestern Polytechnical University (NWPU), Xi'an, Shaanxi, 710072, China*

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ABSTRACT

In practice, an epidemic might be spreading among multi-communities; while the communities are usually intra-connected. In this manuscript, each community is modeled as a multiplex network (i.e., virtual layer and physical one). The connections inside certain community are referred as inter-contacts while the intra-contacts denote the connections among communities. For the epidemic spreading process, the traditional susceptible-infected-recovered (SIR) model is adopted. Then, corresponding state transition trees are determined and simulations are conducted to study the epidemic spreading process in multi-communities. Here, the effect of incorporating virtual layer on the range of individual affected by the epidemic is pursued. As illustrated, multi-summits are incurred if the spreading in multi-communities is considered; furthermore, the disparity between summits varies. This is affected by various factors. As indicated, the incorporation of virtual layer is capable of reducing the proportion of individuals being affected; moreover, disparity of different summits is likely to be increased regarding with scenarios of excluding virtual layer. Furthermore, the summit is likely to be postponed if virtual layer is incorporated.

1. Introduction

The isolated networks are widely investigated to study the relationship between network topology and network activity [1]. Nevertheless, in practice, a network is usually classified into several communities while a community is composed of a group of nodes in the network while the community is more densely connected than with the other nodes of the network. For instance, the Worldwide Airport Network is composed of Airport Network of US, Airport Network of India, Airport Network of China and so forth. Moreover, community detection methods are presented and standards are proposed in order to measure the community partition [2–4]. This indicates most networks are not isolated but interacting with each other, being referred to as a *network of networks* (NoN). Within each individual network, the connectivity links are maintained to represent various connections (often called *inter-contacts*); whereas links are added to connect each network to others (being referred to as *intra-contacts*) [5]. This type of networks is capable of modeling a number of real-world systems, for instance, the European air transport system [6], the global cargo ship network [7], living organisms [8–10] and social networks [11,12].

Recently, researchers started to investigate a special class of NoN, while each node possesses different types of links on different layers [13], being referred to as Multiplex networks. Multiplex networks are widely adopted in various areas, among which, the epidemic spreading process is a rapidly evolving topic [14–16]. In [16,17], a two-layered multiplex network is presented in order to analyze the epidemic spreading process, while the application of this two-layered network model is also thoroughly investigated [18,19]. In this model, the two layers are referred to as a virtual layer and a physical one respectively. The physical layer is composed of individuals while the links represent the corresponding practical body contacts. For the virtual layer, it is consisting of virtual nodes (with each node corresponding to an individual on the physical layer); here, the connections indicate relationships incurred by various social ties, such as phone call, twitter, Wechat and etc. On the virtual layer, the information about the epidemic is spreading among nodes; whereas on the physical layer, biological elements carrying the virus of epidemic are exchanged among individuals through physical contacts.

In order to investigate the epidemic spreading process, various models are presented, for instance, susceptible-infected (SI), susceptible-infected-susceptible (SIS), susceptible-infected-recovered (SIR) and etc. [1,20]. In [21], the SIR model is adopted for the analysis of epidemic spreading process in multilayered networks. While in [14], the SIS model is applied to investigate the epidemic spreading process in multilayered networks. Furthermore, in [22], the SIS model is investigated in multiplex networks using a contact-contagion formulation. While in order to improve the evaluation efficiency, stochastic analysis is performed to pre-

* Corresponding author.
E-mail addresses: ericcan@nwpu.edu.cn (P. Zhu), yangming_g@nwpu.edu.cn (Y. Guo).

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dict the long run behaviors of multiplex networks with the adoption of the proposed stochastic architectures for SIS model in [23]. For this class of disease as H1N1 (Swine Influenza), H5H1 (Avian Influenza) or the SEVERE Acute Respiratory Syndrome (SARS) [24], the favorite approach to describe the spreading process is the SIR model. For SIR model, an infected individual is able to be recovered from an infected state due to the effect of medical treatments or self-rehabilitation.

As discussed previously, a network is composed of a number of communities. Hence, in practice, the epidemic might be spreading among different communities; whereas the communities are usually intra-connected due to the existence of frequent communications among individuals. Thus, the epidemic spreading processes in multi-communities are widely investigated. In [25], an analytical approach is presented and dynamic interactions between two different SIR propagation processes are thoroughly investigated. In [26], the scenario of recurrent epidemics is studied which is an extension from a single community. Furthermore, in [27], the epidemic spreading process is investigated to reproduce the synchronized and mixed outbreak patterns in two communities.

For the epidemic spreading process in multi-communities, there usually exist multi-summits (here, each indicates a larger proportion of individuals being affected by the epidemic). Nevertheless, the summits of the epidemic spreading process in multi-communities are likely to occur at different time. As indicated by previous studies, the disparity of different summits is likely to be affected by a number of factors, such as infectivity probabilities, recovery probability, network topology and etc. Nevertheless, it is still of great interest to investigate the problem of minimizing the range of individuals being affected by the epidemic. Hence, in this manuscript, the evolution process is further investigated which is spreading in two intra-connected communities. Here, the adopted model to mimic the spreading process is the traditional SIR model. Furthermore, each community is characterized by a two-layered multiplex model. Thus, the effect of incorporating virtual layer on the epidemic spreading process is pursued with the analysis of illustrative examples.

Overall, the contributions of this manuscript are summarized as follows:

1. The epidemic spreading process in two communities is investigated while each community is modeled with the adoption of a two-layered multiplex network;
2. The factors that affect the evolution process is thoroughly incorporated, especially the consideration of virtual layer which provides a potential efficient means to control the epidemic spreading process.

The rest of the paper is organized as follows. Section 2 presents some hypothesis used in this manuscript. While the fundamentals of multiplex networks are presented in Section 3; furthermore, the state transition tree is also derived. Then, the construction process of the intra-communities is given in Section 4. Corresponding result and analysis are presented in Section 5. Finally, Section 6 concludes the paper.

2. Assumptions

Here, some assumptions are listed as follows:

1) An individual becomes aware of the epidemic due to communications with neighbors, this occurs with a probability of λ;
2) The infected individual is aware of the epidemic immediately;
3) For the seasonal epidemic, an individual is likely to forget the awareness and becomes unaware again in the future. This happens with a probability of δ;
4) For a susceptible individual, if he or she is aware of the epidemic, then the infectivity is reduced by a factor of γ, as preventive measures are to be taken. Hence, if the original infectivity is β^U, then the reduced probability is $\gamma\beta^U$;
5) The infected individual is recovered with a probability of μ.

3. Methods

3.1. Two-layered multiplex networks

In this manuscript, the epidemic is spreading among multi-communities while each community is indicated by a two-layered multiplex network consisting of a virtual layer and a physical one respectively. The nodes on the virtual layer can exchange information related with the epidemic with each other through virtual contacts; while biological elements carrying the virus are transmitted among individuals through physical contacts. For an illustration, a system consisting of two communities (for instance, A and B) is presented in Fig. 1. As in Fig. 1, *intra-contacts* between the two communities are denoted by dotted lines through which the intra-communication is conducted. For certain community, the connections among nodes on the virtual or individuals on the physical layer might be totally different. Here, the connections in certain layer are represented by corresponding topology.

For the presented model in this work, the information transition process on the virtual layer and epidemic spreading process on the physical layer are relevant processes. As to the virtual layer, if an individual is aware of the epidemic, then preventive measures can be taken which reduces the infectivity by a factor accordingly; this is referred to as self-protection. Furthermore, if an individual on the physical layer is infected, then he or she becomes aware of the epidemic immediately; this indicates the self-awareness ability of the individual.

In order to investigate the epidemic spreading process, the SIR model is adopted in this manuscript. For the physical layer, an individual is anticipated to be in one of the three states, i.e., *susceptible* (**S**) in which the individual is free of the epidemic but might be infected via physical contacts with the infected neighbors, *infected* (**I**) in which the individual is infected by the epidemic and can affect its susceptible individuals, and *recovered* (**R**) in which the individual is recovered from the epidemic through applying medical treatment. For simplicity, the recovered individual is assumed to be immune to the epidemic in this manuscript. Then, the state transition for the individual on the physical layer is derived as $\boldsymbol{S} \rightarrow \boldsymbol{I} \rightarrow \boldsymbol{R}$.

Whereas, for the virtual layer, corresponding node for certain individual is likely to be in one of the following two states, i.e., *unaware* (**U**) in which the individual has no information regarding with the epidemic, *aware* (**A**) in which the susceptible individual is already aware of the epidemic and preventive measures can be taken to protect himself (for instance, reducing the frequency of outside activities, wearing face masks, washing hands frequently to remove the possible virus, taking the vaccination, doing exercises and so forth [28–30]). The individual without awareness will be aware of the epidemic due to the information transition process on the virtual layer; moreover, due to the assumption of seasonal epidemic, an individual with awareness will forget the epidemic. Hence, corresponding transition on the virtual layer is anticipated to be $\boldsymbol{U} \leftrightarrow \boldsymbol{A}$.

Overall, in the proposed model, each individual (including corresponding node on the virtual layer) falls into the following state combinations, i.e., **US, UI, UR, AS, AI** and **AR**. Fig. 2 illustrates the possible transitions between the state combinations of individuals for community A, the outgoing arrow from a given node at time t points to the possible successor state at time step $t+1$ where $X_{PA}Y_{VA}$ means that the state of the individual on the physical layer is in X state and the corresponding node on virtual layer for the in-