Chaos, Solitons and Fractals 107 (2018) 5–12

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Phase-flip in relay oscillators via linear augmentation

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ARTICLE INFO

Article history:
Received 6 June 2017
Revised 28 November 2017
Accepted 6 December 2017

Keywords:
Linear augmentation
Phase-flip
Amplitude death

ABSTRACT

In time-delay coupled relay system of three limit cycle of oscillators, linear augmentation control provides an effective strategy to induce phase-flip from relative phase zero to π between spatially separated oscillators. A coexisting dynamical regime of fixed point and anti-phase synchronization state is harnessed in relay system. Targeting phase-flip transition through linear augmentation is illustrated by using Stuart-Landau oscillators. However, such dynamical transition does not occur in the same system with conjugate coupling through dissimilar variables. Inducing relative phase difference without resorting to variation of internal system parameters and time-delay is important from the view of system control.

1. Introduction

The phenomenon of anti-phase synchronization observed by Christian Huygens in the case of two pendulum clocks hung on a wall was found to be quite remarkable observation in the studies of coupled oscillators [1]. The two oscillators were found to have the same frequency moving opposite in direction with a phase difference of π. Transition from in-phase to anti-phase synchronization state in a system of two coupled oscillators is achieved when time delay is introduced in diffusive coupling [2–4]. Another form of coupling known as conjugate coupling that corresponds to coupling between dissimiliar variables is effective in achieving delay induced phenomenon [5].

In many natural systems, signals between extended systems are transmitted by mechanism of dynamic relay through other systems [6–9]. A simple network motif for relaying signals is a motif that consists of three dynamical systems where two spatially separated systems interact through a central system [7,8]. Since signal transmissions between different systems occur at finite speed, time delays arise naturally in many real systems. Moreover, the presence of feedback loops [10] and aging of interaction pathways between natural systems often resulted in increase of time delay to process information transfer [11]. So, it becomes quite natural to consider delay in the studies of interacting oscillators [12,13] that lies within the intrinsic timescale of the systems [14]. Previous studies have shown that delay could induce a transition in relative phase difference (of two oscillators) from zero to π as a function of the coupling [2] as well as amplitude death [15] in a system of coupled oscillators.

In an earlier work, we have studied the effect of external environment to a relay system of time-delay coupled Hindmarsh–Rose neurons in both chain and star topology [16]. It was observed that a simple direct environmental coupling (linear augmentation) to the central neuron gradually quenched its oscillations, thereby inducing stable synchronization and de-synchronization between outer neurons. This observation opens the possibility of achieving anti-phase synchronization between outer oscillators in relay network by stabilizing fixed point of central oscillator.

In this work, we aim to address the following question. Could we target a fixed point (origin) and anti-phase synchronization state simultaneously in a relay system of coupled limit cycle oscillators? To answer, we consider a system of Stuart-Landau oscillators coupled diffusively with time-delay and study the effect of linear augmentation control. Linear augmentation (LA) is a simple control strategy where a nonlinear system is coupled with a linear system [16,17]. The advantage of this strategy is that the desired state of nonlinear systems can be targeted without recourse to change of internal parameters of the system. The motivation of considering such coupling scheme comes from the pheomenology of many natural systems where the decaying concentration of a system established a diffusive coupling with other self sustaining oscillatory systems [16,18]. Some examples can be drawn from chemical [18] and biological systems [19]. The collective behavior of a population of chemically coupled oscillators is reported [18] where the coupling is maintained through the exchange of chemical species with surrounding common medium. In the case of neuronal systems, it has been shown that the electrical activities of the neurons induced dipole moments in proteins like beta-

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https://doi.org/10.1016/j.chaos.2017.12.004
0960-0779/

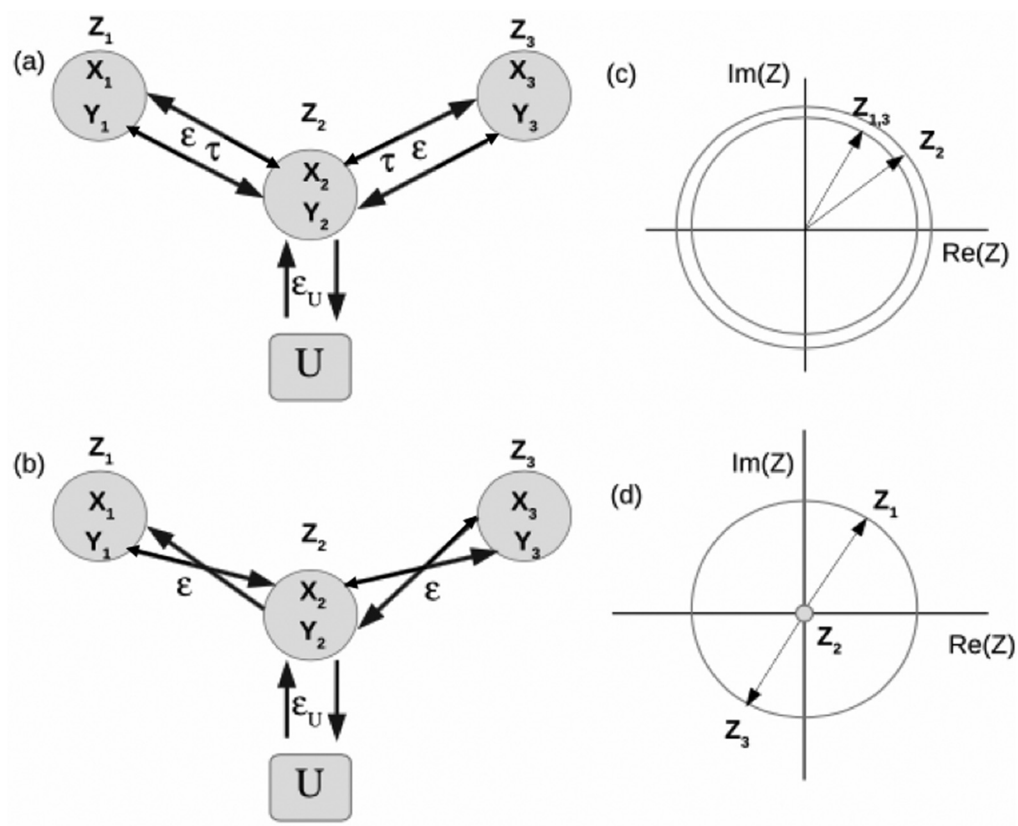

Fig. 1. (a) Schematic diagram of diffusively coupled time-delay relay oscillators with linear augmentation to central oscillator. (b) Linearly augmented relay oscillators with conjugate coupling. (c) Representation of limit cycle oscillators in complex plane where outer oscillators in complete synchronization leads the phase with respect to middle oscillator. (d) Representation of anti-phase synchronization between outer oscillators when origin of middle oscillator is stabilized.

amyloid [19], and these induced activities make a feedback loop to affect the electrical properties of neurons.

Here we consider three birectionally coupled relay oscillators, and investigate scenarios such as how anti-phase schronization arises in the system by employing linear augmentation technique [17]. Here we demonstrate that a relay system of time delay coupled Stuart–Landau limit cycle oscillators supports a dynamical regime where two spatially separated oscillators operate in anti-phase when middle oscillator is driven to its fixed point (origin). Surprisingly, a system of relay limit cycle oscillators with linear augmentation control strategy provides an example where conjugate coupling is not effective to achieve the delay induced dynamical regime.

The paper is organized as follows: An equation of relay Stuart-Landau oscillators with time-delay coupling and a linear augmentation is introduced in Section 2. In Section 3, we extend the study with conjugate coupling to compare the dynamical scenarios arise from the case of time delay coupling. Finally, a summary and discussion are given in Section 4.

2. Time delay coupled Stuart–Landau oscillators with linear augmentation

We consider Landau–Stuart oscillators as the dynamical units of the relay oscillators

$$\dot{Z} = (A + i\omega - |Z|^2)Z \tag{1}$$

where, $Z = x + iy$ is a complex variable, A is a parameter that determines the distance from Hopf bifurcation, and ω is the natural frequency. The system has fixed point at $Z = 0$ (origin), and Hopf bifurcation at $A = 0$. This system settles down to fixed point $Z = 0$ if $A \leq 0$, and it exhibits limit cycle behavior $Z = r\exp(i\omega t)$ if $A > 0$. In the oscillating state, the oscillator have an amplitude $r = \sqrt{A}$ and frequency ω. In the cartesian co-ordinate, the equation becomes

$$\dot{x} = [A - (x^2 + y^2)]x - \omega y \tag{2}$$

$$\dot{y} = [A - (x^2 + y^2)]y + \omega x. \tag{3}$$

A system of time delay coupled relay Stuart–Landau (SL) oscillators with LA coupling to central oscillator is shown in Fig. 1(a). The outer SL oscillators have the following dynamical equations

$$\dot{Z}_{1,3} = (A + i\omega - |Z_{1,3}|^2)Z_{1,3} + \varepsilon(Z_2(t-\tau) - Z_{1,3}(t)) \tag{4}$$

and linearly augmented central oscillator has the following equations

$$\dot{Z}_2 = (A + i\omega - |Z_2|^2)Z_2 + \varepsilon(Z_1(t-\tau) + Z_3(t-\tau) - 2Z_2(t)) + \varepsilon_U U \tag{5}$$

$$\dot{U} = -kU - \varepsilon_U Re(Z_2) \tag{6}$$

where ε is the coupling strength, τ is the time-delay between oscillators, $Re(Z_2)$ is the real part of Z_2, and k is the rate of decay of the linear system U. Such type of coupling scheme has been employed in a system of Hindmarsh–Rose neurons [16] to incorporate an interaction with external environment.

The phase of each oscillator can be evaluated from $\phi_i = tan^{-1}(y_i/x_i)$, and phase difference between oscillators as $\triangle\phi_{ij} = \langle|\phi_i - \phi_j|\rangle$, where the subscripts i and j denote different oscillators and $\langle . \rangle$ denotes the time average. The frequency of each oscillators can also be evaluated as $\Omega = 2\pi/\langle T \rangle$, where $\langle T \rangle$ is the average time-period of successive maxima or minima over an interval of time.

2.1. Symmetry of relay system

We will briefly discuss the symmetry of relay SL oscillators as shown in Fig. 1. With respect to the central oscillator Z_2, the two