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Couple stresses effect on linear instability and nonlinear stability of convection in a reacting fluid

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ABSTRACT

We study the problem of convective movement of a reacting solute in a viscous incompressible fluid occupying a plane layer and subjected to a couple stresses effects. The thresholds for linear instability are found and compared to those derived by a global nonlinear energy stability analysis. In particular, we analyse the effect of no-slip boundary conditions on the stability and instability of convection. The conditions of no-slip at the boundary with couple stresses effect and non constant coefficients which are analysed for the first time in this article.

1. Introduction

In atmospheric physics and oceanographic studies, the convective instability that a top-heavy layer of fluid containing a solute creates has many applications. Furthermore, in pollution, such a solute can cover a city and remain for extended periods of time. Franchi and Straughan [1] introduced a highly nonlinear model for such behaviour, and developed a detailed analysis of its instability.

Independently, Hayat and Nawaz [2] studied a reacting solute with a superimposed magnetic field acting for stagnation point flow in a rotating frame for fluids. Recently, a great deal of interest has focussed on convection in chemically reacting fluids (see Malashetty and Biradar [3], Rahman and Al-Lawatia [4]. In addition, attention has been given to electro-magnetic field effects on such processes (see Kaloni and Mahajan [5] and Maehlmann and Papageorgiou [6]).

Earlier in this work the theoretical and experimental results on the onset of thermal instability (Bénard convection) in a fluid layer under varying assumptions of hydrodynamics, underwent a detailed reviewed, conducted by Chandrasekhar [7]. Such investigations of these kinds of fluids are important, bearing in mind the increasing importance of non-Newtonian fluids in technology and industries. Stokes [8] has put forward the theory of couple-stress fluids. These couple-stresses are present in significant magnitude in fluids with very large molecules. Applications of couple-stress fluid occur in connection with the study of the mechanism of synovial joint lubrication, currently being focussed upon by researchers. A human joint is a dynamically loaded bearing with an articular cartilage as the bearing, and synovial fluid as the lubricant. The normal synovial fluid is clear or yellowish and is a non-Newtonian, viscous fluid. Because of the long chain of lauronic acid molecules found as additives in synovial fluid, Walicki and Walicka [9] modelled the fluid in question as couple-stress fluid in human joints. The issue of a couple-stress fluid and porous medium has been investigated in [10–12]. It should be stressed that these papers consider only the stress-free boundary conditions which assume that there is no mass flux across the boundary. Also, these papers deal with constant coefficients in the system of equations which makes the analysis very poor. The conditions of no-slip at the boundary and nonconstant coefficients in the system, which are believed to be highly relevant in real situations, are (we believe) analysed for the first time in this article.

So, the investigation of the theories of linear instability and nonlinear stability linked to the question of convective movement of a reacting solute in a viscous incompressible fluid occupying a plane layer and subjected to stress effects is this works objective. Analysis of both the linear instability and nonlinear stability thresholds of the governing model is a method of assessing the beginning and type of convection and plays a critical role in comprehending this system. Assessing how suitable linear theory is for predicting the physics of the onset of convection can be achieved by comparison of the thresholds. The highly adaptable energy method [13] is employed so as to establish stability results. Nonlinear energy methods are particularly noted for delimiting the parameters of possible subcritical instability (that area between the linear instability and nonlinear stability thresholds).

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Nomenclature

$(x_1, x_2, x_3) = (x, y, z)$	Cartesian coordinates
$\mathbf{v}$	velocity
p	pressure
c	concentration
$\mathbf{u}$	dimensionless velocity
$\mathcal{P}$	dimensionless pressure
ϕ	dimensionless concentration
g	gravitational acceleration
μ	dynamic viscosity
$\hat{\upsilon}$	couple stress viscosity
Δ	Laplacian
D	solute diffusion coefficient
K_1	chemical reaction rate
P_s	Prandtl number
$\tilde{h}$	plane-tiling planform
$(\bar{v}_i, \bar{p}, \bar{c})$	steady state solution
ρ	density
ρ_0	reference density
c_∞	reference concentration
α_c	salt expansion coefficient
$Ra = R^2$	Rayleigh number
Ra_L	critical Rayleigh number for linear instability theory
Ra_E	critical Rayleigh number for the nonlinear stability theory
a	horizontal wavenumber
a_x	wavenumbers in the x direction
a_y	wavenumbers in the y direction
a_L	critical wavenumber for linear instability theory
a_E	critical wavenumber for the nonlinear stability theory
σ	growth rate
$\vec{\omega} = (\xi_1, \xi_2, \xi_3)$	vorticity vector
$\vec{\psi} = (\psi_1, \psi_2, \psi_3)$	potential vector
Lx	box dimension in the x direction
Ly	box dimension in the y direction

Quantifying the difference between these two thresholds enables assessment of the suitability of linear theory to predict the destabilisation of the double diffusive convection[14–21]. are among the most recent works on the study of convective instabilities in fluid and porous media, while [22–25] have developed and analysed applications for certain convection models.

This paper is organised thus. In the next section, the governing equations of motion and derive the associated perturbation equations will be shown, followed by analyses of linear instability (Section 3) and global nonlinear stability (Section 4) to establish the instability/stability thresholds. The stability analyses involve eigenvalue problems with non-constant coefficients, so a numerical solution to these questions is called for. The appropriate numerical method is explained in Section 5. Section 6, deals with the numerical results for the linear theory and compares them directly with those of the global nonlinear theories.

2. Basic equations

We suppose the fluid is contained in the plane layer $\{z \in (0, d)\} \times \mathbb{R}^2$, and is incompressible, although a Boussinesq approximation is employed in the buoyancy term in the momentum equation. The momentum equation for a fluid containing a solute is then,

$$\rho(v_{i,t} + v_j v_{i,j}) = -p_{,i} + \mu\,\Delta v_i - \hat{\mu}\Delta^2 v_i - \rho\alpha_c k_i g(c - c_\infty), \quad (2.1)$$

where ρ, $\mathbf{v}$, p, c are the constant density, velocity field, pressure, and concentration of solute. Additionally, α_c is the salt expansion coefficient, μ is the dynamic viscosity, $\hat{\upsilon}$ is the couple stress viscosity, g is gravity, c_∞ is a reference concentration, and $\mathbf{k} = (0, 0, 1)$. Throughout, we use standard indicial notation and the Einstein summation convention so that e.g. $v_{i,t} = \partial v_i/\partial t$, and $p_{,i} = \partial p/\partial x_i$, $v_j v_{i,j} \equiv (\mathbf{v}.\nabla)\mathbf{v}$, and Δ is the Laplacian. The balance of mass equation is

$$v_{i,i} = 0 \quad (2.2)$$

The equation governing the evaluation of the solute concentration is, cf., Hayat and Nawaz [2],

$$c_{,t} + v_i c_{,i} = D\Delta c - K_1(c - c_\infty). \quad (2.3)$$

Here D is the solute diffusion coefficient, and K_1 is the chemical reaction rate, the chemical reaction being represented by the term $K_1(c - c_\infty)$.

The equations for our model can be reduced as:

$$\begin{aligned} v_{i,t} + v_j v_{i,j} &= -\frac{1}{\rho}p_{,i} + \nu\Delta v_i - \hat{\upsilon}\Delta^2 v_i - k_i g\alpha_c(c - c_\infty), \\ v_{i,i} &= 0, \\ c_{,t} + v_i c_{,i} &= D\Delta c - K_1 c. \end{aligned} \quad (2.4)$$

where $\nu = \mu/\rho$ and $\hat{\upsilon} = \hat{\mu}/\rho$.

The boundary conditions to be satisfied are no-slip at the boundaries $z = 0$ and $z = d$ with the concentrations fixed there. Thus,

$$\begin{aligned} &v_i = 0, \quad \text{on} \quad z = 0, d; \\ &c = c_U, \quad z = d; \qquad c = c_L, z = 0; \end{aligned} \quad (2.5)$$

where c_U, c_L are constants with $c_U > c_L$.

We then find there is a steady solution $(\bar{v}_i, \bar{c}, \bar{p})$ whose stability we wish to examine, and this is

$$\begin{aligned} \bar{v}_i &\equiv 0, \\ \bar{c} &= [\frac{c_U - c_L\cosh(A_1 d)}{\sinh(A_1 d)}]\sinh(A_1 z) + c_L\cosh(A_1 z), \end{aligned} \quad (2.6)$$

where $\bar{p}$ may then be found from (2.4), and where A_1 is given by

$$A_1^2 = \frac{K_1}{D}. \quad (2.7)$$

Next, we drive perturbation equations to this steady state. Hence, put $v_i = \bar{v}_i + u_i$, $c = \bar{c} + \phi$, $p = \bar{p} + \mathcal{P}$, and employ the scales

$$\tau = \frac{d^2}{\nu}, \quad U = \frac{\nu}{d}, \quad P = \frac{\rho\,\nu U}{d}, \quad L = d,$$

where τ, U, L, P are time, velocity, length, and pressure scales. Define $\xi = A_1 d = (\sqrt{K_1/D})d$, and pick the concentration scale $C^\sharp$ as

$$C^\sharp = U\sqrt{\frac{\nu\,\Delta C}{D\alpha_c g d}}$$

where $\Delta C = c_U - c_L > 0$. Furthermore, define the salt Rayleigh number R^2 as

$$R^2 = \frac{\alpha_c g d^3\,\Delta C}{D\,\nu}, \quad (2.8)$$

and the salt Prandtl number as $P_s = \nu/D$. We also need the non-dimensional numbers η and M where

$$\eta = \frac{c_L}{c_U - c_L}, \quad (2.9)$$