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Fractional-differential approach to the study of instability in a gas discharge

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ABSTRACT

Using an approach based on the kinetic equation of fractional order on the time variable, two types of instability in a gas discharge are investigated: the instability of the electron avalanche and the sticking instability in a nonself-maintained discharge.

1. Introduction

Progress in the study of the gas discharge physics under high pressure is largely determined by knowledge of the physical properties of the discharge. Especially, this corresponds the initial stage, which in gases and liquids is accompanied by the generation and propagation of specific ionization waves. The wide practical application of gas discharge various forms stimulates studies of their spatial structure.

In [1], the initial stage of the development of the ionization wave front instability due to the multiplication of electrons of low background density is considered. An expression for the growth rate of small perturbations is found. It is shown that the propagation front is unstable with respect to small perturbations forming protrusions or dips. The growth rate of the instability can be defined as a function of the reduced field strength that is universal for a given gas.

In [2], the microstructure of the current channel was experimentally detected in the breakdown of homogeneous air gaps by voltage pulses of the nanosecond range in electric fields insufficient to form of a streamer. As a mechanism for the formation of a microstructure, the development of the instability of the ionization process in the avalanche stage is proposed, that leads to the formation of a self-similar spatial structure. It is shown that the microstructure of streamer discharges in homogeneous gaps can also be explained within the framework of the proposed model.

There is a large number of both experimental and theoretical studies on the stability of ionization fronts. Nevertheless, there is no unified theory of the development of instability. The latter is due to the complexity of accounting for all important factors affecting processes. Thus, the development of new effective approaches in this field of research is extremely important.

Today, in the study of complex systems, an analytical approach based on the use of the mathematical apparatus of fractional integro-differentiation is of great interest. At present, it can be considered established that the mathematical apparatus of integro-differentiation of fractional order [3-7] adequately expresses the fundamental physical concepts underlying the physics of systems with deterministic chaos, allowing to take into account in a natural way spatial and temporal non-locality and features of fractional-order geometry. The use of this apparatus makes it possible to interpret with great accuracy complex experimental data for such phenomena as anomalous diffusion [8], heat transfer in media with a complex structure [9], dispersion transport in semiconductors [10], calculation of thermodynamic properties of surfaces [11], etc.

In the works on the use of fractional analysis known to date, various interpretations of fractional derivatives are given, starting from differentiation in a space with fractional dimension to the description of non-stationary processes and anomalous transport phenomena. There are various critical points of view about such interpretations. We have previously proposed our interpretation in [6,7].

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In [6,7], using a variety of approaches, a number of phenomena in quantum-statistical systems described by fractional-order equations were considered. The main emphasis is on the physical nature of the occurrence of fractional derivatives in the equation. It turned out that the introduction of fractional derivatives in the equation for the Green's function is analogous to the introduction of the inter-particle interaction. As a result, the equation was obtained in a state analogous to the van der Waals equation. Further, we generalized the well-known method of statistical physics of introducing the interaction parameter into the system. We have obtained a whole class of various systems with a Hamiltonian involving the interaction, which can be described by means of a fractional-differential approach. Such an approach can be very convenient, because it is often impossible to describe systems with a Hamiltonian involving interaction by ordinary methods. Further, in our paper [7], in the framework of the Hartree–Fock approximation, the inverse problem is considered when, according to the known form of the fractional-differential equation, the inter-particle interaction potential was obtained, which can lead to such a form. Thus, the appearance of fractional derivatives in the equations describing the many-particle system we associate with the inter-particle interaction.

Finally, we note following. As is well known, the turbulent state is natural for a plasma. In this state, the plasma obeys the laws of anomalous diffusion [12-16]. In such a state, the mean free paths of particles are power functions [16]. This can lead to a fractional equation for the distribution function (details see in [8]).

It should be noted that fractional derivatives cannot be used on fractal since it cannot be considered a linear (vector) space. A mathematically correct approach is given in the books [17,18].

Using an approach based on the kinetic equation of fractional order on the time variable, two types of instability in a gas discharge are investigated: the instability of the electron avalanche and the sticking instability in a nonself-maintained discharge (Fig. 1).

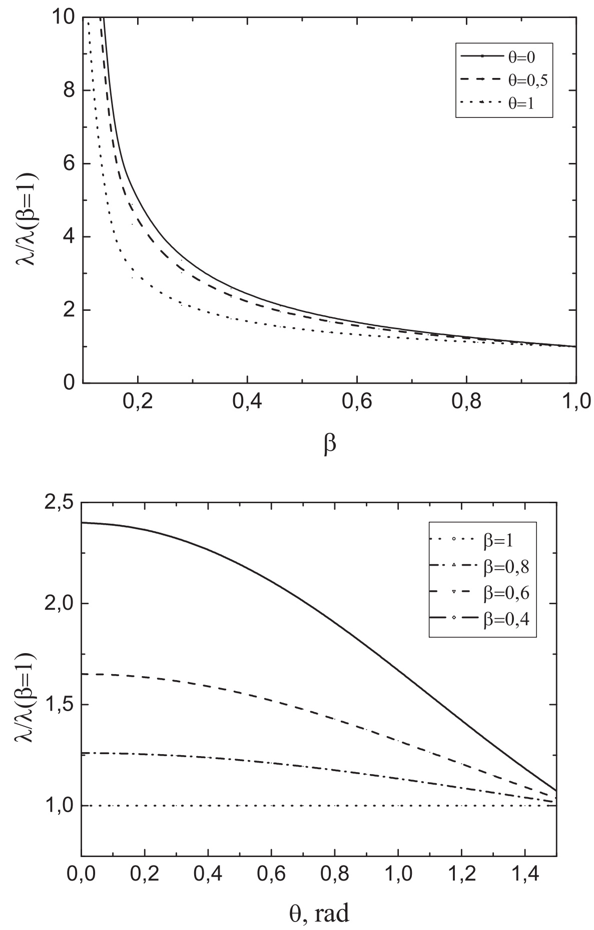

Fig. 1. Dependence of the instability parameter: a) on the order of the fractional derivative for different values of the angle θ, b) on the angle θ for different values of the order of the fractional derivative.

2. Instability of electron avalanche in gas discharge

A standard approach to investigating the stability of an avalanche is based on the use of the kinetic equation for the electron concentration in the avalanche (an adiabatic approximation is used when ion motion is neglected)

$$\frac{\partial n}{\partial t} = \alpha \upsilon_{dr} n - \upsilon_{dr}\frac{\partial n}{\partial z} + D_e \Delta n. \tag{1}$$

where α is the Townsend ionization coefficient, υ_{dr} is the electron drift velocity, D_e is the electron diffusion coefficient, the axis Oz is directed along the field. In Eq. (1) we pass to the Riemann-Liouville fractional derivatives [3]

$$\partial^{\beta}_{-\infty x} f(x) = \frac{1}{\Gamma(1-\{\beta\})}\frac{\partial^{[\beta]+1}}{\partial x^{[\beta]+1}}\int\limits_{-\infty}^{x}\frac{f(\xi)d\xi}{(x-\xi)^{\{\beta\}}}. \tag{2}$$

where $[\beta]$ is the integer part of β and $0 \le \{\beta\} < 1$ is the fractional part of β. (Note that the fractional derivative (2) is also called the Liouville derivative [19]). Then

$$\frac{1}{t_0}\partial^{\beta}_{-\infty\tau} n = \alpha \upsilon_{dr} n - \upsilon_{dr}\frac{\partial n}{\partial z} + D_e \Delta n, \tag{3}$$

where $\tau = t/t_0$ is a dimensionless time, t_0 is the some characteristic time. The problem that solving in the present paper is a rare case when the parameter t0 is not included in the final result. In general, as t0 one can use, for example, the characteristic ionization time. We will be sought the solution in a standard way in a following form: $n \propto \exp(-i\omega\tau + ikr)$. Then[1]

$$\frac{1}{t_0}(-i\omega)^{\beta} = \alpha \upsilon_{dr} - i\upsilon_{dr} k_z - D_e k^2, \tag{4}$$

where k=$2\pi/\lambda$, λ is a characteristic size of the disturbance inhomogeneity. Next, we note that

$$(-i\omega)^{\beta} = |\omega|^{\beta}\exp\left(-i\beta\frac{\pi}{2} + i\beta\arctan\frac{\mathrm{Im}\omega}{\mathrm{Re}\omega}\right). \tag{5}$$

Then

$$|\omega|^{\beta}\cos\beta\left(\arctan\frac{\mathrm{Im}\omega}{\mathrm{Re}\omega} - \frac{\pi}{2}\right) = t_0\left(\alpha\upsilon_{dr} - D_e k^2\right)$$

$$|\omega|^{\beta}\sin\beta\left(\arctan\frac{\mathrm{Im}\omega}{\mathrm{Re}\omega} - \frac{\pi}{2}\right) = -t_0 \upsilon_{dr} k_z$$

The last equations give

$$\mathrm{Im}\omega = t_0^{\frac{1}{\beta}}\left(\left(\alpha\upsilon_{dr} - D_e k^2\right)^2 + \upsilon_{dr}^2 k_z^2\right)^{\frac{1}{2\beta}}\cos\left(\frac{1}{\beta}\arctan\frac{\upsilon_{dr} k_z}{\alpha\upsilon_{dr} - D_e k^2}\right) \tag{6}$$

The condition for the onset of instability is the following inequality

$$\mathrm{Im}\ \omega \ge 0. \tag{7}$$

[1] We used the property of the fractional Riemann-Liouville derivative: $\partial^{\beta}_{-\infty x} e^{ax} = a^{\beta} e^{ax}$.