Chaos, Solitons and Fractals 106 (2018) 5–15

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

A novel hyperchaotic system with infinitely many heteroclinic orbits coined

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ARTICLE INFO

Article history:
Received 13 June 2017
Revised 18 August 2017
Accepted 27 October 2017
Available online 22 November 2017

MSC:
34C45
34C37

Keywords:
5D Hyperchaotic system
Ellipse-parabola-type and hyperbola-parabola-type heteroclinic orbits
Singularly degenerate heteroclinic cycle
Lyapunov function
ω–Limit set and α–limit set

ABSTRACT

Based on the famous Shimizu–Morioka system, this paper proposes a novel five-dimensional Shimizu–Morioka-type hyperchaotic system that has an infinite set of heteroclinic orbits. Of particular interest are the following observed properties of the system: (i) the existence of both ellipse-parabola-type and hyperbola-parabola-type of equilibria; (ii) the strange attractor coexisting either non-isolated equilibria or two pairs of symmetrical equilibria; (iii) the existence of the proposed strange attractors and hyperchaotic attractors bifurcated from the corresponding singularly degenerate heteroclinic cycles; (iv) the existence of an infinite set of both ellipse-parabola-type and hyperbola-parabola-type heteroclinic orbits.

1. Introduction

Chaos, as a phenomena that displays intrinsic randomicity, is widely distributed in nature and displays a broad range of applications, such as medical science [1,2], time series prediction [3], biology [4], secure communication [5] and so on. It is known that the introduction of Lorenz system [6] established the chaos theory as a main branch of contemporary science. A great many other chaotic systems [7–12], i.e. neighboring systems of Lorenz system, demonstrate the strange attractor with a positive Lyapunov exponent as the Lorenz system, expanding in one direction. In contrast, hyperchaos, which was first proposed by Rössler in 1979 [13], characterizes the strange attractor with more than one positive Lyapunov exponent, expanding in at least two different directions and showing more complex dynamical behaviors. Therefore, hyperchaos can be employed in more wide fields than chaos.

In past decades, many researchers have become involved in detecting singular orbits from nonlinear systems with either self-excited or hidden chaotic and hyperchaotic attractors. Refering to [14–46] and the references therein, most of those chaotic and hyperchaotic systems have periodic solutions, limit cycles, homoclinic and heteroclinic orbits, singularly degenerate heteroclinic cycles and so on.

In chaos and hyperchaos theory, the singular orbits of a dynamical system are extremely important for revealing some natures of its complex dynamical behaviors. This can be explained by taking singularly degenerate heteroclinic cycles for most chaotic systems [14–18,27,28] for example. Referring to those systems, the collapse of infinitely many singularly degenerate heteroclinic cycles is one of important routes to chaos. Recently, many chaotic systems are constructed with line, curve and even plane of equilibria [47–50]. Namely, infinitely many equilibrium points yield the possibility of infinitely many singular trajectories, such as homoclinic and heteroclinic orbits, the aforementioned singularly degenerate heteroclinic cycles and so on. As a result, it is a demanding and challenging task to generate new hyperchaotic systems with some of aforementioned infinitely many singular trajectories, especially the heteroclinic orbits.

Based on the famous Shimizu–Morioka system [8] and by the techniques of replication and mutation of chaos [51,52], the present paper introduces a new five-dimensional (5D) hyperchaotic system, finding it to have infinitely many heteroclinic trajectories. This new hyperchaotic system, which is found to display hyperchaotic attractor with either a line of equilibria or two pairs of symmetrical equilibria, is explored through theoretical analysis and numerical simulation. Meanwhile, infinitely many singularly de-

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https://doi.org/10.1016/j.chaos.2017.10.029
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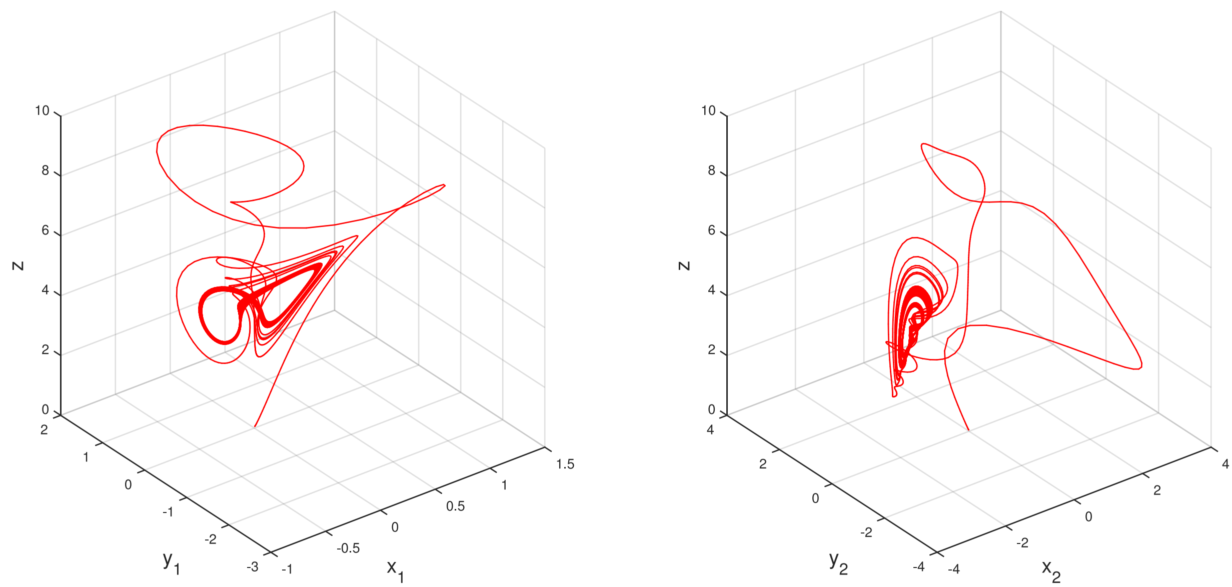

(a) x_1-y_1-z (b) x_2-y_2-z

Fig. 2.1. Phase portrait of the system (2.2) in projection spaces (a) $x_1 - y_1 - z$, (b) $x_2 - y_2 - z$ with parameters $(c_1, d_1, e_1, c_2, d_2, e_2, b, a_1, a_2) = (4, -1, -1, 7, -1, -1, 0.1, 5, 0.01)$ and initial values $(x_1^0, y_1^0, x_2^0, y_2^0, z^0) = (1.618, 3.14, 2.718, 4.6692, 0.618) \times 1e-2$.

generate heteroclinic cycles are computed together with the corresponding hyperchaotic attractors near them, which may be a route to hyperchaos as the case of chaos for most chaotic systems. In particular, we have rigorously proved that there exists an infinite set of both ellipse-parabola-type and hyperbola-parabola-type heteroclinic orbits.

The rest of this paper is organized as follows. In Section 2, the new 5D hyperchaotic system is introduced. Section 3 analyzes the local stability of the new proposed system. By the aid of numerical simulation, Section 4 coins the singularly degenerate heteroclinic cycles and the corresponding hyperchaotic attractors near them. In Section 5, one rigorously proves that the new system has no homoclinic orbits but infinitely many heteroclinic orbits. Finally, some conclusions are drawn in Section 6.

2. Novel 5D hyperchaotic system

The famous 3D Shimizu–Morioka system [8] is described by

$$\begin{cases} \dot{x} = y, \\ \dot{y} = x - \lambda y - xz, \quad \alpha, \lambda > 0, \\ \dot{z} = -\alpha z + x^2. \end{cases} \tag{2.1}$$

and has been intensively studied in past three decades [14,42–46] from viewpoints of singular trajectories, bifurcations, kneadings, symbolic dynamics, normal forms, bi-parametric structures, etc.

Motivated by the replication and mutation of chaos in [51,52], one in this paper derives the following 5D hyperchaotic model by replicating and mutating the Shimizu–Morioka system (2.1)

$$\begin{cases} \dot{x_1} = y_1, \\ \dot{y_1} = -y_1 + c_1 x_1 + d_1 x_1^3 + e_1 x_1 z, \\ \dot{x_2} = y_2, \\ \dot{y_2} = -y_2 + c_2 x_2 + d_2 x_2^3 + e_2 x_2 z, \\ \dot{z} = -bz + a_1 x_1^2 + a_2 x_2^2, \end{cases} \tag{2.2}$$

where x_i, y_i and z are state variables, and $a_i, b, c_i, d_i, e_i \in \mathbb{R}$, $i = 1, 2$ are constant parameters. When $e_1 e_2 = 0$, the investigations for this 5D system are reduced to those for a 3D system, which is not our interest. So, in the sequel, we always assume that $e_1 e_2 \neq 0$.

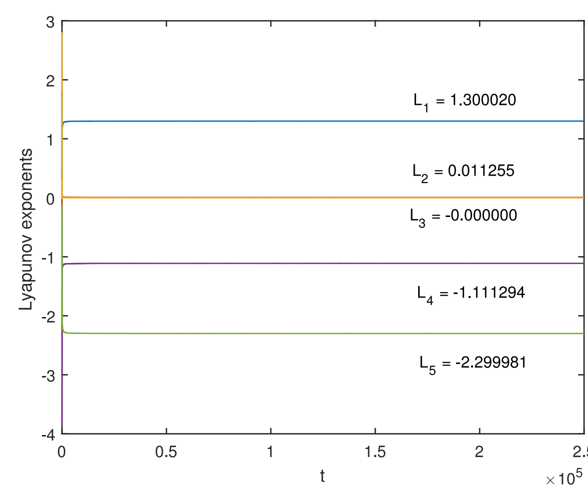

Fig. 2.2. Lyapunov exponents of the system (2.2) for $(c_1, d_1, e_1, c_2, d_2, e_2, b, a_1, a_2) = (4, -1, -1, 7, -1, -1, 0.1, 5, 0.01)$ and $(x_1^0, y_1^0, x_2^0, y_2^0, z^0) = (1.618, 3.14, 2.718, 4.6692, 0.618) \times 1e-2$.

Set the initial condition

$$\left(x_1^0, y_1^0, x_2^0, y_2^0, z^0\right) = (1.618, 3.14, 2.718, 4.6692, 0.618) \times 1e-2.$$

For

$$(c_1, d_1, e_1, c_2, d_2, e_2, b, a_1, a_2) = (4, -1, -1, 7, -1, -1, 0.1, 5, 0.01),$$

numerical simulation displays that the system (2.2) has a hyperchaotic attractor with the Lyapunov exponents

$$L_1 = 1.300020, L_2 = 0.011255, L_3 = -0.000000,$$
$$L_4 = -1.111294, L_5 = -2.299981,$$

depicted in Fig. 2.1–2.2.

When

$$(c_1, d_1, e_1, c_2, d_2, e_2, b, a_1, a_2) = (4, -1, -1, 2, -1, 2, 0.0, 6, -1),$$