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Dynamical analysis of a novel autonomous 4-D hyperjerk circuit with hyperbolic sine nonlinearity: Chaos, antimonotonicity and a plethora of coexisting attractors

G.D. Leutcho [a,b], J. Kengne [a,*], L. Kamdjeu Kengne [b]

[a] Unité de Recherche d'Automatique et Informatique Appliquée (LAIA), Department of Electrical Engineering, IUT-FV Bandjoun, University of Dschang, Dschang, Cameroon

[b] Unité de Recherche de Matière Condensée, d'Electronique et de Traitement du Signal(LAMACETS), Department of Physics, University of Dschang, P.O. Box 67, Dschang, Cameroon

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ABSTRACT

In this paper, a novel fourth-order autonomous hyperjerk circuit is proposed and the corresponding dynamics is systematically analyzed. Two anti-parallel semiconductor diodes form the nonlinear component necessary for chaotic oscillations. The mathematical model of the novel circuit consists of a fourth-order ("elegant") autonomous hyperjerk system with (a single) hyperbolic sine nonlinearity. The fundamental dynamic properties of the model are investigated including fixed points and stability, phase portraits, bifurcation diagrams, and Lyapunov exponent plots. Period-doubling bifurcation, periodic windows, coexisting bifurcations, symmetry recovering crises, and antimonotonicity (i.e. concurrent creation and annihilation of periodic orbit) are reported when monitoring the systems parameters. One of the main findings in this work is the presence of various windows in the parameter space in which the novel 4D-hyperjerk system develops the interesting property of multiple coexisting attractors (e.g. coexistence of two, three, four, five, six, seven height or nine disconnected periodic and chaotic attractors). To the best of the authors' knowledge, this striking phenomenon is unique and has not yet been reported previously in a hyperjerk circuit, and thus represents a significant contribution to the understanding of the behavior of nonlinear dynamical systems in general. Laboratory experiments of the oscillator are carried out to verify the theoretical analysis.

1. Introduction

In Newtonian mechanics, the three first derivatives of the position (dx/dt, d^2x/dt^2, d^3x/dt^3) are called velocity, acceleration and jerk respectively. Jerk systems represent a subclass of three-dimensional autonomous dynamic systems with the interesting feature of mathematical elegance [1-3]. One of the most interesting features of jerk systems is the possibility of physical implementation by using off-the-shelf electronic components (e.g. resistors, capacitors, diodes, op. amplifiers). From the extremely rich literature deserved to jerk systems (and circuits), we know that such types of systems can experience various forms of nonlinear behaviors including period doubling, chaos, periodic windows, intermittency, crisis, antimonotonicity, hysteresis, coexisting bifurcations, and multistability, just to name a few [3-8]. The mathematical model of a jerk system is an explicit third-order ordinary differential equation (ODE) describing the time evolution of a single scalar variable x:

$$\frac{d^3x}{dt^3} = J\left(\frac{d^2x}{dt^2}, \frac{dx}{dt}, x\right) \quad (1)$$

Where J is called the "Jerk". By extension, a fourth order system in the form $d^4x/dt^4 = f(d^3x/dt^3, d^2x/dt^2, dx/dt, x)$ is called "hyperjerk system" or "snap system" [9,10,12]. The fourth time derivative d^4x/dt^4 is described as "jouce", "sprite" or "surge" but is generally referred to as "snap" in the relevant literature. Recently, the study of high order dynamic systems in general and hyperjerk systems in particular has become one of the most followed research avenue. There are numerous elegant examples of flows in higher-dimensional spaces, many of which are important because they better model phenomena in the real world, which typically involve many interacting variables. Such systems can be obtained by adding variables to known existing three-dimensional chaotic systems, by coupling multiple low-dimensional systems, or by simply scanning the larger parameter space of such systems for chaotic at-

* Corresponding author.
E-mail address: kengnemozart@yahoo.fr (J. Kengne).

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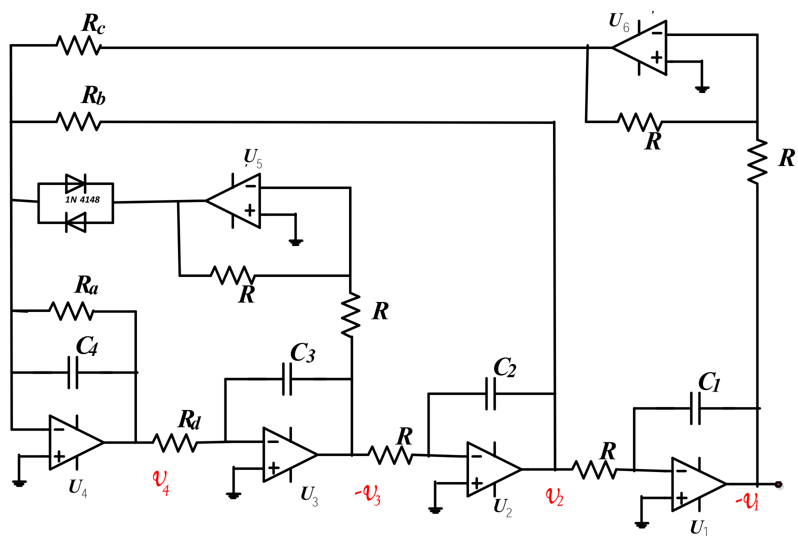

Fig. 1. Simple electronic circuit realization of the novel Jerk system with hyperbolic sine nonlinearity. The hyperbolic nonlinearity of the model is implemented by a pair semiconductor diodes connected in antiparallel. The following values of electronic circuit components are used for the analysis: $R_a = 5.555\,\text{k}\Omega$, $C_1 = C_2 = C_3 = C = 10\,\text{nF}$, $R_b = 3.333\,\text{k}\Omega$, $R_d = 2.985\,\text{k}\Omega$, $R_c - tuneable$, a pair of general purpose signal diodes $D_1 = D_2 = 1N4148$ ($\eta = 1.9$, $V_T = 26\,\text{mV}$, $I_S = 2.682\,\text{nA}$), $U_j\,(j = 1, 2, 3, 4) = TL084CN$.

tractors. The challenging issue is to increase the dimension while maintaining the elegance and to display the high-dimensional dynamics in a meaningful and comprehensive manner. In retrospect, Clouverakis and Sprott [9] have considered hyperjerk systems from a mathematical or dynamical systems view point focusing on their formal elegance and have numerically demonstrated that surprising simple functional forms of hyperjerk systems already exhibits a rich variety of complicated dynamical behaviors. Particularly, they paid special attention on potential chaotic and/or hyperchaotic behavior and the interesting routes to chaos leading to it as well as their usefulness for the identification of relatively simple chaotic 4D dynamical systems. In Ref. [10], Linz discusses the connection between externally driven nonlinear oscillators and specific uni- and bidirectional coupled systems of two autonomous oscillators; thus providing an interesting reinterpretation of simple chaotic forms of hyperjerk systems. The possible cases of equivalence between four dimensional autonomous dynamical systems and hyperjerk dynamics have been investigated by Zeraoulia and Sprott [11]. The authors showed that a wide class of four-dimensional vector fields possesses this property. Five new elementary chaotic snap flow and a generalization of an existing flow are presented in [12] through an extensive numerical search. Ref. [13] describes a relatively simple hyperjerk system with an exponential nonlinearity. The physical implementation of the new hyperjerk system is carried out based on FPAA (Field Programmable Analog Array) technology. Very recently, based on the hyperchaotic systems proposed by Clouverakis and Sprott [9], novel four order hyperchaotic hyperjerk where designed on investigated both numerically and experimentally [14-16].

Motivated by the above mentioned work, this contribution proposes a simple autonomous RC snap circuit with complex dynamics but with an extremely simple nonlinear part. The novel circuit represents the extension to fourth order of a recently studied jerk circuit by Kengne and collaborators [17]. The novel snap circuit consists of Op. Amps, resistors, capacitors, and a pair of semiconductor diodes connected in antiparallel to synthesize the hyperbolic (sine) nonlinearity of the model. This nonlinearity accounts for the complex and striking features observed in the proposed circuit such as multistability (e.g. the occurrence of up to nine disconnected coexisting attractors in the phase space) and antimonotonicity as well. To the best of author's knowledge, the proposed circuit represents the simplest snap circuit with such complex dynamics reported to date [9-16]. Also, the goal in this paper is to make a contribution on the study of the dynamics of the proposed circuit by addressing the following key issues: (a) to explain the chaotic mechanism of the new circuit based on an appropriate mathematical model; (b) to develop some tools such as bifurcation diagrams that can be exploited for a rigorous design of the oscillator under investigation; (c) to point out some of its unknown and striking features like antimonotonicity and coexistence of several attractors in the phase space; (d) to carry out an experimental study of the system to validate results of theoretical analysis. The overall motivation behind this work is to enrich the literature of nonlinear oscillations in part, and provide appropriate documentation that can be used for practical technological exploitations of such type of oscillators as well.

The rest of the paper is organized as follows: Section 2 deals with the modeling process. The electronic structure of the oscillator is described and a suitable mathematical model is derived to investigate the dynamics of the system. Section 3 is concerned with the numerical analysis. Various bifurcation diagrams combined with their corresponding graphs of numerically computed Lyapunov exponents are plotted to illustrate different routes leading to chaos. The occurrence of plethora of attractors (up to nine coexisting solutions for the same parameters set) is also discussed using bifurcation diagrams as arguments. In addition the bubbling phenomenon is presented. The laboratory experimental results of the oscillator are carried out in Section 4. Some concluding remarks are presented in Section 5.

2. Description and analysis of the model

2.1. Circuit description

The schematic diagram of the proposed fourth order hyperjerk circuit is depicted in Fig. 1. The circuit is made up of four successive integrators in a multiple feedback loop in addition to a nonlinear (acceleration) feedback loop consisting of a negative gain amplifier with a pair of anti-parallel semiconductor diodes (D_1, D_2). The circuit proposed in this work can be viewed as an electronic analog (analogue computer) of Eq. (19) defined by Buncha and Banlue [12] where the exponential nonlinearity is replaced by a hyperbolic sine function. Alternatively, the novel circuit represents the extension to fourth order of a recently studied jerk