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Dynamics of ac-driven sine-Gordon equation for long Josephson junctions with fast varying perturbation

Zamin Gul, Amir Ali*, Irshad Ahmad

Department of Mathematics University of Malakand Chakdara Dir (L), KPK, Pakistan

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ABSTRACT

A long Josephson junction comprising regions with phase discontinuities driven by an external ac-drive is studied. An inhomogeneous sine-Gordon equation is used, that depicts the dynamics of long Josephson junctions with phase discontinuities. Perturbation technique along with asymptotic analysis and the method of averaging are applied to obtain an average dynamics in the form of double sine-Gordon equations for both small and large driving amplitudes. From the obtained average dynamics, it is determined that, the external ac-drive may affect the presence of the ground state of the junction. Specifically, the cases for $0-\kappa$ and $0-\pi-0$ junctions are discussed. In the presence of an external ac-drive, the critical facet length b_c is analyzed for $0-\pi-0$ junction above which the ground state is non-uniform. The critical bias current γ_c for $0-\kappa$ junction is investigated, at which the junction switches to a resistive state. Further, the interaction of localized defect modes and the fast oscillating drive is studied for both $0-\kappa$ and $0-\pi-0$ junctions, which is explained by using Lagrangian approach. Numerical simulations are performed to support our analytical calculations.

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1. Introduction

In the recent century, an impressive consideration has been given to the study of Josephson junctions. Josephson junctions are important solid-state tools on which experimental analysis can be carried out with relative clarity. In 1962, Brian Josephson predicted theoretically that, when two superconductors are brought close together with a thin layer of an insulator in between them, electrons could "tunnel;; through the non-superconducting barrier from the one superconductor to the other. It is due to the quantum mechanical waves in the two overlapped superconductors. The tunneling of electrons is called dc-Josephson effect and the device is named after its discovery as Josephson junction [1]. There have been many applications of Josephson junctions in several fields, for example, data-processing systems and microwave oscillators. The precise investigation of Josephson junctions gives many facts on the stability of numerous physical systems, such as, ferromagnetic systems, charge-density waves and anti-ferromagnetic systems.

If we represent a Josephson phase by $\psi(x, t)$ then $I = I_c \sin\psi(x,t)$ is called current phase relation. A novel behaviour in Josephson junctions was first investigated by Bulaevskii et al. [2]. In experiments, it was observed that Josephson vortex transporting a fraction of magnetic flux quantum. This inconceivable phenomenon could be performed by intrinsically constructing piecewise fixed phase discontinuity $\Theta(x)$. This transformed the supercurrent relation into $I = I_c \sin(\psi + \Theta)$. Various experimental methods are performed to create nonlinearity in the Josephson phase [3–5]. These include abrikosov vortex [6], connection of magnetic impurities [7], pair of injectors [8] and multi-layer junctions with control thickness over the ferromagnetic barrier [9–11]. Josephson junctions with phase discontinuity have promising applications in information processing and data storage [12,13].

In ideal long Josephson junctions, the phase difference $\psi(x, t)$ satisfies a sine-Gordon equation. The sine-Gordon equation arises extensively in the study of nonlinear systems, because of its multisoliton solutions, solitary wave solutions and periodic solutions. The elementary nonlinear confined solutions of the sine-Gordon model can be separated into two main parts, that is, *kink* and *breather*. The 'kink' predicts the static and dynamic properties of localized excitations, while 'breather' has a solution that oscillates in time and localized in space and has frequency lying under the linear band. The sine-Gordon model predicts different physical phenomena, that is, the oscillation of localized modes in plasma physics [14], pattern configuration [15], stochastic propagation [16] and information transport in microtubules [17].

Here, we consider the spatially perturbed sine-Gordon equation

$$\psi_{tt} - \psi_{xx} + \sin(\psi + \Theta) = \gamma + \alpha\,\psi_t + f\cos(\Omega t), \tag{1}$$

* Corresponding author. Tel: +92 345 812 5077.
E-mail address: amiralishahs@yahoo.com (A. Ali).

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to study long Josephson junctions with ac-drive. Here ψ is the variance between the phases of the wave functions, γ is a value of bias current and α is a damping parameter. The constant f is the amplitude of oscillating ac-drive and Ω is driving frequency of the system. Because of the nondimensionalization of the temporal variable t, the Josephson plasma frequency corresponds to $\Omega = 1$. A theoretically relevant study of Eq. (1) for $\Omega < 1$ has been analyzed previously [18–20]. Here, we study an experimentally relevant case $\Omega \gg 1$. The phase-shifts configurations $\Theta(x)$ considered herein are

$$\Theta(x) = \begin{cases} \pi, & |x| \le b, \\ 0, & |x| > b, \end{cases} \tag{2}$$

$$\Theta(x) = \begin{cases} 0, & x < 0, \\ \kappa, & x > 0, \end{cases} \tag{3}$$

which are known as $0-\pi-0$ and $0-\kappa$ junctions respectively. For physically meaningful solution, we use the continuity conditions

$$\psi(\pm b^{+}, 0) = \psi(\pm b^{-}, 0), \qquad \psi_x(\pm b^{+}, 0) = \psi_x(\pm b^{-}, 0), \tag{4}$$

and

$$\psi(\pm 0^{-}) = \psi(\pm 0^{+}), \qquad \psi_x(\pm 0^{-}) = \psi_x(\pm 0^{+}), \tag{5}$$

for $0-\pi-0$ and $0-\kappa$ junctions. The unperturbed sine-Gordon equation together with a phase shift (2) has a uniform ground state $\psi_0 = 0\,(\mod 2\pi)$. The stability analysis shows that the ground state is spatially non-uniform for the constant solution when the critical facet length $b_c > \pi/4$, and the solution is unstable [21]. Similarly, for the undriven Eq. (1) together with phase shift (3), the ground state of the system is [29]

$$\psi_0(x,t) = \begin{cases} 4\tan^{-1}\exp(x_0 + x), & x < 0, \\ \kappa - 4\tan^{-1}\exp(x_0 - x), & x > 0, \end{cases} \tag{6}$$

with $x_0 = ln(\tan\kappa/8)$, describes fractional fluxon that is spontaneously generated at the point of discontinuity.

Here, we apply a perturbation technique together with multiple-scale expansion and the method of averaging to develop "average" equations representing the dynamics of long Josephson junctions with phase shifts. The obtained equations are double sine-Gordon equations that represent the slowly varying dynamics over the fast oscillating ac-drive of the original considered equation. Similar average dynamics were obtained before by using a "normal form" technique [22], where many canonical transformations to the Hamiltonian system are applied and convert "mean-zero" to a higher order. Similarly, Fourier series along with asymptotic expansion was applied to split the phase ψ into the sum of slowly and fast-varying fragments and the coefficients were obtained in the form of Bessel functions of the first kind [23,24]

$$j_1 = J_0(a_1) + \frac{a_1^2\alpha^2(J_2(a_1) - J_0(a_1))}{4\Omega^2} + \frac{a_1\alpha^2 J_1(a_1)}{\Omega^2}, \tag{7}$$

$$j_2 = \frac{J_1^2(a_1)}{\Omega^2} + \frac{a_1^2\alpha^2 J_0(a_1)J_2(a_1)}{32\Omega^4} + \frac{a_1\alpha^2 J_1(a_1)J_2(a_1)}{16\Omega^4}, \tag{8}$$

with $a_1 = -f/\Omega^2$. In this article, we use simple scaling parameters to obtain the coefficients in simple explicit relations. A simple and very useful analytical approach of the sine-Gordon equation can also be seen, where the authors have derived the current-voltage characteristics and study the fluxon dynamics in long Josephson junctions driven by a spatially non-uniform bias current density and flux flow oscillators with spatially inhomogeneous driving [25–27].

For the non-zero amplitude ($f \neq 0$), the critical facet length b_c and the critical bias current γ_c are analyzed for $0-\pi-0$ and $0-\kappa$ junctions. We show that by increasing the amplitude of the external fast varying drive, the critical facet length b_c increases, while the critical bias current γ_c decreases. This is another part of the article. We also study the interaction of localized mode and the fast oscillating ac-drive. We use the Euler-Lagrange approximations to discuss the localized modes in long Josephson junctions in an infinite domain.

The present paper is organized as follow. In Sections 2 and 3, we consider the fast oscillating ac-drives and presenting our analytical approach to obtain the gradually-varying dynamics in the form of "averaged" equations. We use asymptotic analysis together with multi-scale expansions and the method of averaging to obtain the average dynamics, which describe the double sine-Gordon equation. In Sections 4 and 5, by considering the obtained averaged equations, we examine analytically the critical facet length b_c and the value of an applied bias current γ_c for both the junctions. We show that, the threshold distance in $0-\pi-0$ junctions increases by the fast external ac-drive, while the critical current decreases in $0-\kappa$ junctions. In Sections 6 and 7, we use a Euler-Lagrange approach to discuss the localized modes in long Josephson junctions. Section 8 is dedicated to numerical simulation to confirm our analytical results and also compare our results with the existing literature. Finally Section 9 conclude the paper.

2. Averaging of sine-Gordon equation with a large driving amplitude

In this section, we calculate an average nonlinear system that illustrates the dynamics of Eq. (1). This system has been approximated before by using different approaches [23,24,28]. We use a simple scaling $f = h/\epsilon^{5/2}$, with $h \sim \mathcal{O}(1)$. We consider a rapidly oscillating ac-drive with a small parameter $\epsilon = 1/\Omega^{2/3} \ll 1$. As the considered Eq. (1) depends both on the fast time-scale $t = \mathcal{O}(\epsilon)$ and $t = \mathcal{O}(1)$, so we define

$$\tau_n = \epsilon^{n/2} t, \quad n \in \mathbb{Z}, \quad n \ge -3. \tag{9}$$

We further seek the asymptotic expansion in the form

$$\psi(x,\tau) = \psi_0 + \epsilon^{1/2}\psi_1 + \epsilon^{1}\psi_2 + \epsilon^{3/2}\psi_3 + \cdots, \tag{10}$$

where $\psi_j = \psi_j(x, \tau_{-3}, \ldots)$ and $\tau_{-3} = t/\epsilon^{3/2}$. Here τ_{-3} is assumed to be a fast variable. Over the fast-time scale, it is also assumed that the average

$$\langle \psi_i \rangle = \frac{1}{p}\int_0^p \psi_i(x, \tau_{-3}, .)d\tau_{-3} = 0, \tag{11}$$

that is, ψ_i for $i \in \mathbb{Z}^{+}$ are periodic in fast variable τ_{-3} with $p = 2\pi$. The supposition considered above is productive because any arbitrary ψ_i which is independent of the fast variable τ_{-3} can be merged in ψ_0 and so $\psi_0(x, \tau_{-2}, \ldots)$ is known as the average of $\psi(x,t)$. Plugging expansion (10) in Eq. (1), we obtain the following hierarchy of equations

- $\mathcal{O}(1/\epsilon^3)$ $\quad \mathcal{D}_{-3}^2\psi_0 = 0, \quad (12)$

integrating twice, we obtain $\psi_0 = C_1(\tau_{-2}, \ldots)\tau_{-3} + C_2(\tau_{-2}, \ldots)$. To obtain ψ_0 is independent of τ_{-3}, we take a simple choice of $C_1(\tau_{-2}, \ldots) = 0$. Hence we conclude that $\psi_0 = \psi_0(x, \tau_{-2}, \ldots)$.

- $\mathcal{O}(1/\epsilon^{5/2})$ $\quad \mathcal{D}_{-3}^2\psi_1 + 2\mathcal{D}_{-3}\mathcal{D}_{-2}\psi_0 - h\cos(\tau_{-3}) = 0. \quad (13)$

Simplifying and integrating twice, we obtain $\psi_1(x, \tau_{-3}, \ldots) = -h\cos(\tau_{-3})$, which shows that ψ_1 is a function of fast time scale τ_{-3} and is independent of slow time scales τ_n, $n \in \mathbb{Z}$, $n \ge -2$.

- $\mathcal{O}(1/\epsilon^2)$ $\quad \mathcal{D}_{-3}^2\psi_2 + 2\mathcal{D}_{-3}\mathcal{D}_{-2}\psi_1 + \mathcal{D}_{-2}^2\psi_0 + 2\mathcal{D}_{-3}\mathcal{D}_{-1}\psi_0 = 0, \quad (14)$

simplifying and averaging over the period p, we obtain the solvability condition

$$\mathcal{D}_{-2}^2\psi_0 = 0, \tag{15}$$

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