Chaos, Solitons and Fractals 107 (2018) 111–127

Contents lists available at ScienceDirect

Chaos, Solitons and Fractals

Nonlinear Science, and Nonequilibrium and Complex Phenomena

journal homepage: www.elsevier.com/locate/chaos

Existence and global exponential stability of pseudo almost periodic solution for neutral delay BAM neural networks with time-varying delay in leakage terms

Chaouki Aouiti[a], Imen Ben Gharbia[a], Jinde Cao[b,c,*], Mohammed Salah M'hamdi[d], Ahmed Alsaedi[e]

[a] University of Carthage, Faculty of Sciences of Bizerta, Department of Mathematics, Research Units of Mathematics and Applications UR13ES47, BP W, 7021 Zarzouna, Bizerta, Tunisia
[b] School of Mathematics, and Research Center for Complex Systems and Network Sciences, Southeast University, Nanjing, 210 096, China
[c] Department of Mathematics, Faculty of Science, King Abdulaziz University, Jeddah, 21589, Saudi Arabia
[d] Department of Technology, Faculty of Technology, University of Bejaia, Bejaia, 06000, Algeria
[e] Nonlinear Analysis and Applied Mathematics (NAAM) Research Group, Department of Mathematics, King Abdulaziz University, Jeddah, 21589, Saudi Arabia

ARTICLE INFO

Article history:
Received 28 December 2016
Revised 4 July 2017
Accepted 21 December 2017

Keywords:
Pseudo almost periodic solution
Bidirectional associative memory (BAM) neural networks
Mixed delays
Leakage terms
Global exponential stability

ABSTRACT

In this paper, bidirectional associative memory (BAM) neural networks with time-varying delays in leakage terms are investigated. A set of sufficient conditions are obtained for the existence and the exponential stability of pseudo almost periodic solutions for this class of neural networks by applying Banach's fixed point theorems, and differential inequality techniques. Finally, we present a numerical example and simulation to illustrate the effectiveness of the theoretical results. We extend and improve previously know results.

1. Introduction

Bidirectional Associative Memories (BAM) have been proposed as models of dynamical neural networks which have been extensively studied and widely used in many research fields such as pattern recognition and signal processing [1]. BAM are able to develop attractors allowing the network to perform various types of recall under noisy conditions and successfully carry out pattern completion. Many advances were made since Kosko introduced the BAM in 1988 [2–21].

As is well known, time delays are often encountered unavoidably in many practical systems such as biological, artificial neural networks, automatic control systems, population models, and so on. More specifically, the delays occur in the communication and response of neurons owing to the finite switching speed of amplifiers in the electronic implementation of analog neural networks. The delay is a source of instability and oscillatory response of the networks. Therefore, the study of the stability and convergent dynamics with delays has raised considerable interest in recent years [22–37].

In addition, a particular attention has been paid towards the stability analysis of neural networks and dynamic systems involving time-delay in the leakage (or forgetting) term [16,38–59]. The author in [16] discussed the problem of bidirectional associative memory (BAM) neural networks with constant delays in the leakage term

$$\begin{cases} \dot{x}_i(t) = -a_i(t)x_i(t-\alpha_i) \\ \qquad\qquad + \sum\limits_{j=1}^{m} a_{ji}(t)f_j(y_j(t-\tau_{ji})) + I_i(t), \quad i = 1, \cdots, n, \\ \dot{y}_j(t) = -b_j(t)y_j(t-\beta_j) \\ \qquad\qquad + \sum\limits_{i=1}^{n} b_{ij}(t)h_i(x_i(t-\zeta_{ij})) + J_j(t) \quad j = 1, \cdots, m. \end{cases} \tag{1}$$

Moreover, Yongkun Lin and Youqin Li [38] considered neutral delay BAM neural networks with time-varying delays in leakage

* Corresponding author.
E-mail address: jdcao@seu.edu.cn (J. Cao).

https://doi.org/10.1016/j.chaos.2017.12.022
0960-0779/

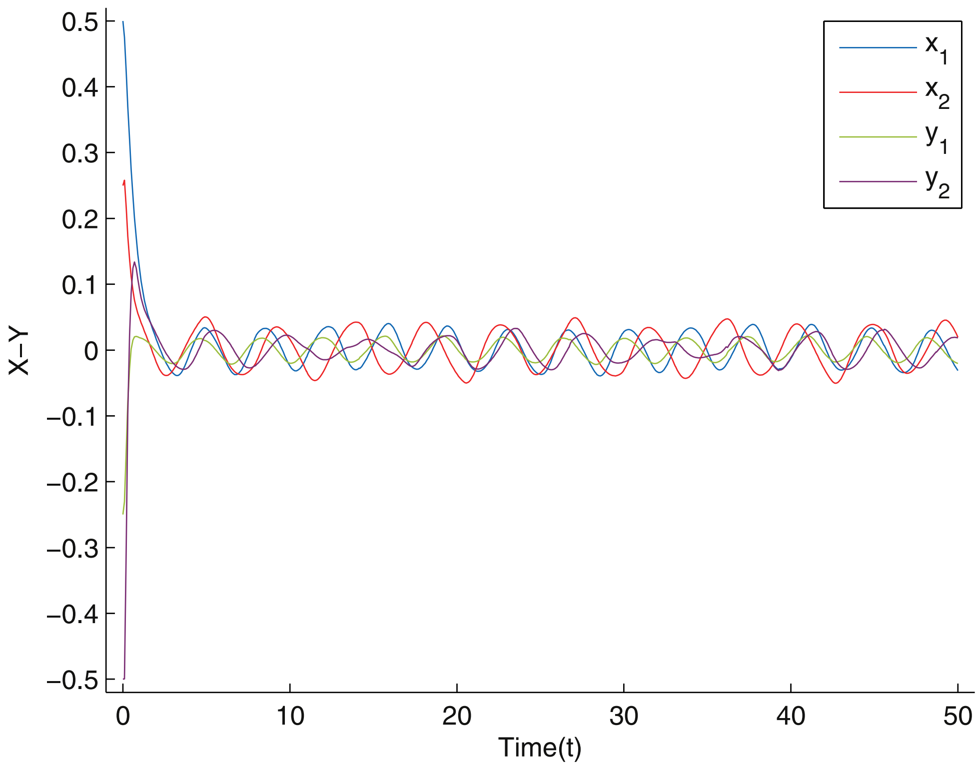

Fig. 1. Numerical solution $Z(t)=(x_1(t),x_2(t),y_1(t),y_2(t))^T$ of system (3) for initial value $\phi(t)=(0.5,0.25,-0.25,-0.5)^T$.

terms of the following form:

$$\begin{cases}\dot{x}_i(t) &= -a_i(t)x_i(t-\alpha_i(t))+\sum_{j=1}^{m}a_{ji}(t)f_j(y_j(t-\tau_{ji}(t)))\\ &\quad+\sum_{j=1}^{m}p_{ji}(t)f_j^1(y_j'(t-\sigma_{ji}(t)))+I_i(t),\quad i=1,\cdots,n,\\ \dot{y}_j(t) &= -b_j(t)y_j(t-\beta_j(t))+\sum_{i=1}^{n}b_{ij}(t)h_i(x_i(t-\zeta_{ij}(t)))\\ &\quad+\sum_{i=1}^{n}q_{ij}(t)h_j^1(x_i'(t-\varsigma_{ij}(t)))+J_j(t)\quad j=1,\cdots,m.\end{cases}\tag{2}$$

In real-world applications, however, equations with a distributed delay provide generally a more realistic description for models of mathematical biology and in particular the population dynamics. As we all know, many phenomena in nature have oscillatory character and their mathematical models have led to the introduction of certain classes of functions to describe them. Such a class form pseudo almost periodicity, which is the main subject of this paper, was introduced in the literature in the early nineties by Zhang [50]. In the context of differential equations, pseudo almost periodic solutions, which are more general and complicated than periodic and almost periodic solutions, were studied in [51–58].

To the best of our know ledge, up to now, there are no papers published on the pseudo almost periodic solution to neutral delay BAM neural networks with time-varying delays in the leakage term.

Motivated by the discussion above, in this paper, we investigate the existence, uniqueness and global stability of pseudo almost periodic solution for neutral type neural networks with the leakage term

$$\begin{cases}\dot{x}_i(t) &= -a_i(t)x_i(t-\alpha_i(t))+\sum_{j=1}^{m}a_{ji}(t)f_j(y_j(t-\tau_{ji}(t)))\\ &\quad+\sum_{j=1}^{m}p_{ji}(t)f_j^1(y_j'(t-\sigma_{ji}(t)))\\ &\quad+\sum_{j=1}^{m}c_{ji}(t)\int_0^{+\infty}K_{ji}(s)g_j(y_j(t-s))ds\\ &\quad+I_i(t),\qquad i=1,\cdots,n,\\ \dot{y}_j(t) &= -b_j(t)y_j(t-\beta_j(t))+\sum_{i=1}^{n}b_{ij}(t)h_i(x_i(t-\zeta_{ij}(t)))\\ &\quad+\sum_{i=1}^{n}q_{ij}(t)h_j^1(x_i'(t-\varsigma_{ij}(t)))\\ &\quad+\sum_{i=1}^{n}d_{ij}(t)\int_0^{+\infty}N_{ij}(s)l_i(x_i(t-s))ds\\ &\quad+J_j(t)\qquad j=1,\cdots,m.\end{cases}\tag{3}$$

where $t\in\mathbb{R}$, n, m are the number of neurons in layers, $x_i(t)$ and $y_j(t)$ denote the activations of the ith neurone and the jth neuron at time t; a_i and b_j represent the rate with which the ith neuron and jth neuron will reset their potential to the resting state in isolation when they are disconnected from the network and the external inputs at time t; f_j, g_j, f_j^1, h_i, h_i^1 and l_i are the activation functions; α_i, β_j, τ_{ji}, σ_{ji}, ζ_{ij} and ς_{ij} are transmission delays at time t; a_{ji}, p_{ji} c_{ji} are elements of feedback templates at time t and b_{ij}, q_{ij} and d_{ij} are elements of feed-forward templates at time t; I_i, J_j denote biases of the ith neuron and the jth neuron at time t, $i=1,2,\cdots,n$, $j=1,2,\cdots,m$.

Several methods can be used to study the existence of pseudo almost periodic solution of the system (3) since the activation functions can be any one of the many types such as hyperbolic, Lipschitz, monotone, non-differentiable, unsaturated etc. As examples, we can note the contraction mapping theorem, Brouwer's fixed point theorem, Leggett-Williams fixed point theorem [63],