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Steady state thermoelastic contact problem in a functionally graded material

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Abstract

This paper is concerned with the stationary plane contact of a functionally graded heat conducting punch and a rigid insulated half-space. The frictional heat generation inside the contact region due to sliding of the punch over the half-space surface and the heat radiation outside the contact region are taken into account. Elastic coefficient μ, thermal expansion coefficient α_t and coefficient of thermal conductivity k are assumed to vary along the normal to the plane of contact. With the help of Fourier integral transform the problem is reduced to a system of two singular integral equations. The equations are solved numerically. The effects of nonhomogeneity parameters in FGMs and thermal effect are discussed and shown graphically.

Keywords: Functionally graded material; Fourier transform; Frictional heating; Heat radiation; Singular integral equation; Fredholm integral equation

1. Introduction

The importance of contact problems in solid mechanics stems from the very basic fact that load application to deformable bodies are mainly done through contact between two bodies. There are of course exceptions; such as loading of the boundary by fluid pressure or various kinds of body forces such as gravitational or magnetic forces. When two solids are in contact, the determination of the state of stress in the media has been the subject of study in literature for many years and the problems are usually termed as contact problems. Usually the contact problems are of two types:

(a) the bodies in contact are bonded together and consequently, the contact regions are known a priori, and the main task is to determine the stress distribution in the media;

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(b) the bodies are in contact without bond so that the region of contact is not known. In such class of problems determination of the contact region (which depends upon geometric features of the bodies as well as upon the load distribution on the system) becomes an additional task. Due to the application of load the area of contact may increase, decrease or may even remain stationary. Accordingly, contact problems are classified as advancing, receding or stationary. Contact problems in which frictional forces at the contact surface are not taken into account have been studied by many investigators. Among several works done we may mention a few: Comez et al. [8], Chaudhuri and Ray [5], El-borgi et al. [9], Jing and Liao [17], Fabrikant [12], Barik et al. [4], Avci and Yapici [2]. Another factor plays a vital role in the study of contact problem. If the surfaces of the solid in contact are not smooth, and the contact surface changes due to relative motion of the solids, there will be generation of heat due to frictional effects at the contact region. The generated heat produces significant thermoelastic distortion of the contacting surfaces, which in turn, can effect the contact pressure distribution. The notable points in this kind of problem are (i) moving source of heat (ii) normal and tangential loadings between the solids and (iii) the unknown contact area. Problems of this type have been discussed by many investigators [1,3,6,7,13,14,34,36,15] considering solids as homogeneous elastic material.

But over the last few decades various problems in solid mechanics are being studied where the elastic coefficients are no longer constants but they are function of position. The investigations result from the fact that idea of nonhomogeneity in elastic coefficients is not at all hypothetical, but more realistic. Elastic properties in soil may vary considerably with positions. The earth crust itself is nonhomogeneous. Besides these, some structural materials such as functionally graded materials (FGMs) have distinct nonhomogeneous character. For example, in graded composite materials, graded regions are treated as series of perfectly bonded composite layers, each layer being assigned slightly different properties. In FGMs the material properties vary gradually with location within the body. In many applications FGMs are found to be better substitutes for conventional homogeneous materials. Among several uses of FGMs, one such is the use of FGMs in automotive brakes and clutches [10,20,35,37,38] where the effect of frictional heat generation is the subject of concern to the scientists. When brakes are applied to a moving system, the kinetic energy produced at the wheel is transformed into heat energy, which does not dissipate fast enough into the air stream from the brake surface into the brake disk and as a result, the high temperatures and thermal stresses that accompany them produce a number of disadvantageous effects such as surface cracks or permanent distortions. The thermal effect also affects the contact pressure between the surfaces. In order to avoid such type of damage FGMs have been considered as protecting coatings between the contact surfaces. From various studies on contact problems [16,18,23] it has been observed materials with controlled gradients in mechanical properties of compositions and structures exhibit resistance to contact deformation and damage that cannot be realized in conventional homogeneous materials. The conventional ferrous material used in a friction brake and clutch causes thermoelastic instability. A coated layer of properly graded ceramic metal FGMs has the advantage of the heat and corrosion resistance of ceramic and the mechanical strength of metal, associated with thermal efficiency of the system and bonding strength along the coating substrate interfaces. The use of FGM layer would reduce the magnitude of residual and thermal stresses. The feasibility of FGMs on the brake and clutch system has been investigated by Jang and Ahn [16].

FGMs made from ceramics and metals are also suitable for use in high temperature generated systems, for example, the use of ceramic composite microstructure in gas-turbines can protect metals and improve the life and reliability of the thermal barrier coatings. FGMs formed by appropriately combining two or more materials in a perfectly designed manner can be shown to be more resistant to crack initiation and propagation [11]. FGMs are used as interfacial zones to reduce stresses arising from matching different material properties, to increase bonding strength and to provide protection against thermal and chemical environment [21]. In some special environments where additional load bearing capacity is needed, properly designed FGMs could be useful.

Study of thermoelastic problems in a functionally graded media is quite involved because of the fact that in addition to the complicated form of the basic equations due to arbitrary variations of elastic moduli with position, the heat conduction equation too becomes complicated when thermal conductivity coefficient is position dependent. Basically due to these constraints analytical study of thermoelastic problems in functionally graded media has been restricted to problems with special types of variation of elastic and thermal moduli.